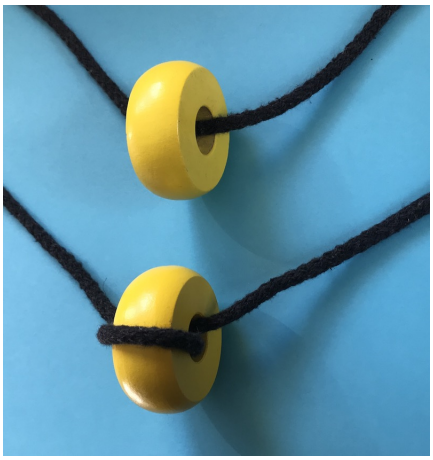
Lesson 2

A-Level Applied Mathematics
Mechanics : Statics : Year 2

2.1 Getting Knotted

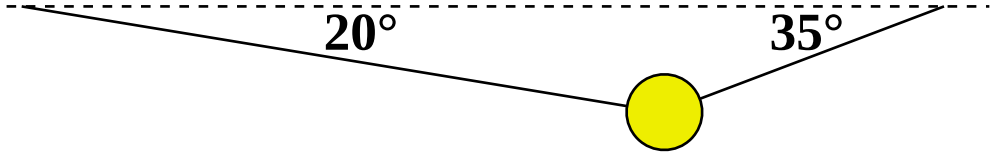
When a bead is threaded onto a string, if the bead is described as smooth, then the tension in the string on one side of the bead is the same as on the other side. However, the bead can be knotted onto the string in which case the tensions in the string on either side may be different. In Lesson 1, the beads were all smooth but in this Lesson the emphasis will be on beads that are knotted onto the string.



An unknotted (smooth) and knotted bead threaded onto string

2.2 Hanging Around

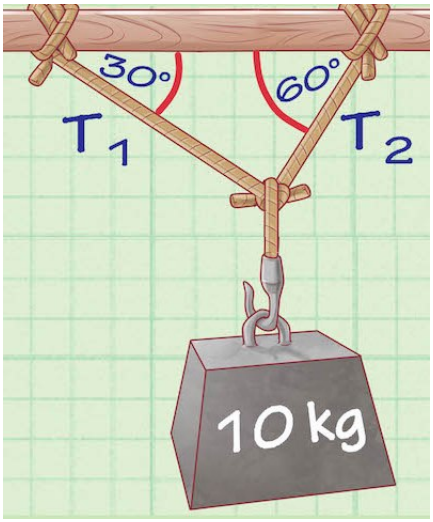
A light metal ring is knotted onto a light, inextensible string and a mass of 4 kg hangs vertically downwards from the ring. The string makes an angle of 20° to the horizontal on one side of the ring and an angle of 35° to the horizontal on the other, as shown.



Find the tensions T_{20} and T_{35} in the string.

2.3 Exercise

Question 1



A 10 kg mass is suspended from a knot in a spanning rope, each end of this in turn being tied to a wooden beam, as shown. The system is in equilibrium.
The spanning rope makes angles of 30° and 60° to the horizontal.

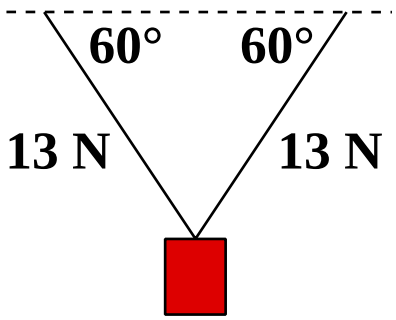
(i) By balancing forces horizontally at the knot, show that,

$$T_1 \cos 30 = T_2 \cos 60$$

(ii) Determine the magnitude of the tensions T_1 and T_2

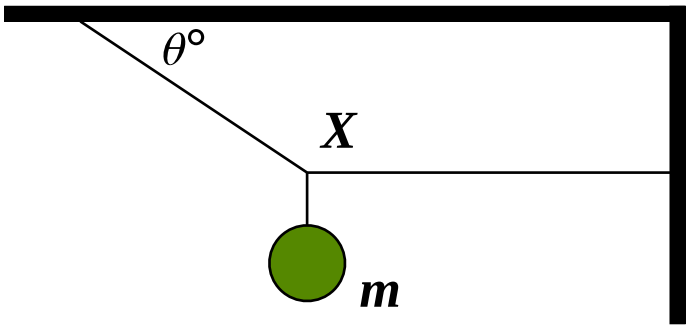
Question 2

The diagram shows an object in equilibrium hanging by two ropes, each under 13 N of tension. The object is knotted onto the two ropes which are light and inextensible.



One of the two ropes is cut.
Eventually the system settles once again into a state of equilibrium,
What is the tension in the one remaining rope ?

Question 3



The figure shows a block of mass *m* hanging at rest.
The light inextensible wire fastened to the wall is horizontal and has a tension of 29 N.
The wire fastened to the ceiling is also light and inextensible.
It has a tension of 38 N and makes an angle θ with the ceiling.

(i) Explain how you know that there is a knot at the point *X*, where the mass is connected to the spanning wire, rather than some frictionless arrangement.

(ii) Determine, in degrees, the size of angle θ

(iii) What is the mass, in kilograms, of *m* ?

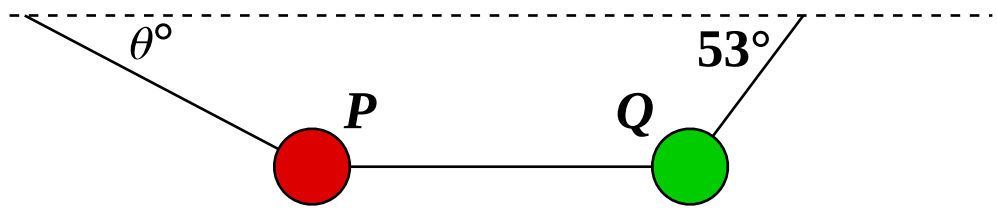
Question 4

Two blocks *P* and *Q*, of 2 kg and 4 kg respectively, are tied onto a light inextensible string. The string between them is horizontal.

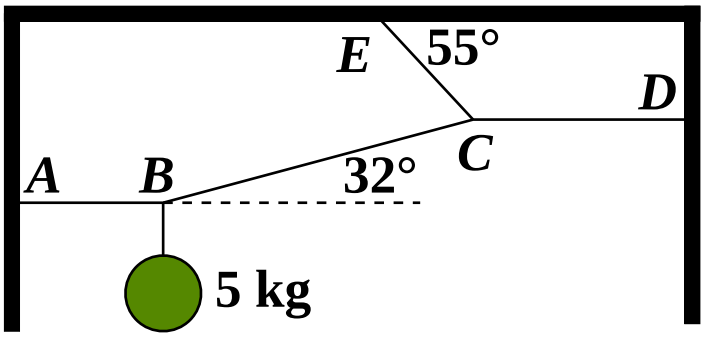
On the other side of the 4 kg block the string makes an angle of 53° to the horizontal.

On the other side of the 2 kg block the string makes an angle of θ to the horizontal.

Given that the system is in equilibrium, determine the angle, θ



Question 5



A 5 kg mass is suspended from the network of light inextensible strings shown. Strings *AB* and *CD* are horizontal, and strings *BC* and *CE* are at 32° and 55° to the horizontal respectively. The strings are joined by firm knots at *B* and *C*. Calculate the tension in each of the four strings, T_{AB}, T_{BC}, T_{CD} and T_{CE}