Lesson 3

A-Level Applied Mathematics
Mechanics : Statics : Year 2

3.1 Limiting equilibrium

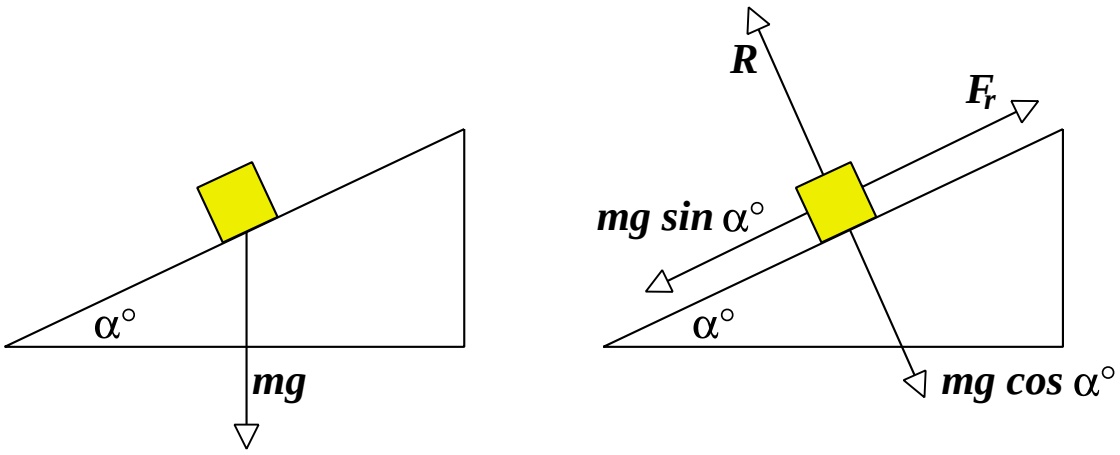
Consider the situation of a particle at rest on an incline.
In such situations the friction force that is calculated from $F_{r\,max} = \mu R$ is the maximum friction force that can be present without movement commencing.
So,

$$0 \leqslant F_r \leqslant F_{r\,max}$$

We are mostly interested in a particle that is on the verge of moving - in limiting equilibrium - and for such a particle F_r takes on the maximum value.

3.2 How to find μ in "real life"

To find μ for a book on a table, tip the table until the book just starts to slide.



Perpendicular to the slope;

$$R = mg\cos\alpha$$

Parallel to the slope;

$$F_r = mg\sin\alpha$$

Link;

$$F_r = \mu R$$
$$mg\sin\alpha = \mu\, mg\cos\alpha$$
$$\sin\alpha = \mu\cos\alpha$$
$$\mu = \frac{\sin\alpha}{\cos\alpha}$$
$$\mu = \tan\alpha$$

So μ is simply $\tan\alpha$.

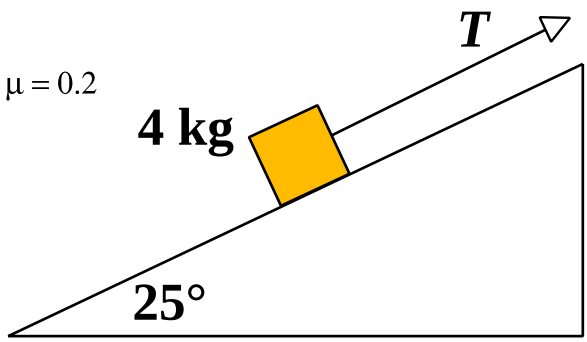
Angle α is easy enough to measure, and $\tan\alpha$ can then be found from a calculator.
Notice that we did not need to know the weight of the book.

This is a special situation in which no force, other than gravity, acted on the book.

3.3 When more than "just gravity" acts

A box of mass 4 kg is on the verge of sliding down a rough slope, inclined at 25° to the horizontal, when restrained by a rope as shown.

The coefficient of friction between box and slope is 0.2



(i) What is the weight of the box ?

(ii) The weight is to be resolved into component parts.

(a) What is the component parallel to the slope ?

(b) What is the component perpendicular to the slope ?

(iii) Draw a good sized diagram showing all forces acting on the block except for weight.

In place of the weight, insert your two part (ii) answers.

(iv) Calculate the normal reaction, R.

(v) Calculate the friction force, F_r.
Is F_r acting up or down the slope ?

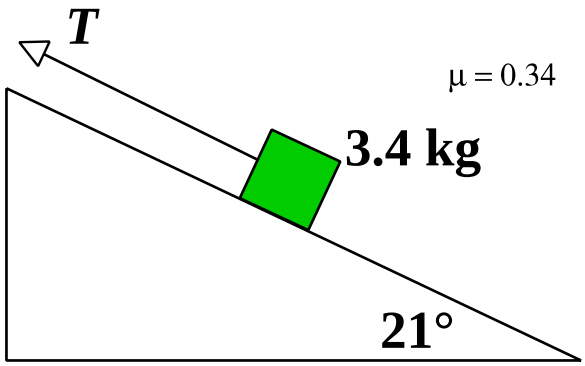
(vi) Find the tension in the restraining rope.

3.4 Exercise

In this exercise, where required, take the value of g as 9.8 m s^{-2}

Question 1

A box of mass 3.4 kg is on the verge of sliding **UP** a rough slope, inclined at 21° to the horizontal, when pulled by a rope as shown.
The coefficient of friction between box and slope is 0.34



(i) What is the weight of the box ?

(ii) The weight is to be resolved into component parts.

(a) What is the component parallel to the slope ?

(b) What is the component perpendicular to the slope ?

(iii) Draw a good sized diagram showing all forces acting on the box except for weight.
In place of the weight, insert your two part (ii) answers.

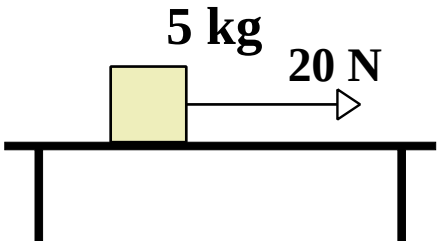
(iv) Calculate the normal reaction, R.

(v) Calculate the friction force, F_r.
Is F_r acting up or down the slope ?

(vi) Find the tension in the pulling rope.

Question 2

A heavy book of mass 5 kg is acted on by a horizontal force of 20 N.
The coefficient of friction, μ, between book and coffee table is 0.42

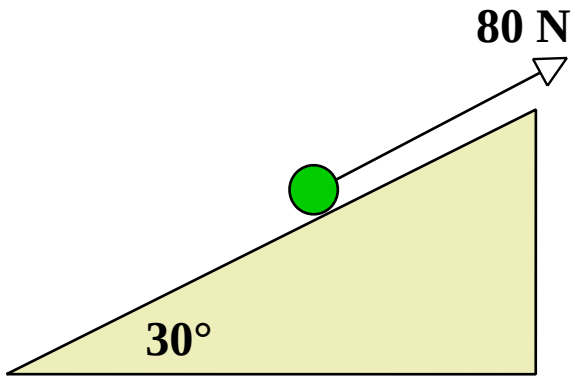


(i) Calculate the normal reaction.

(ii) Calculate the maximum possible friction force that could become available to prevent motion, and hence state if the 20 N force will cause the book to move or not.

Question 3

A particle of mass 12 kg is at rest on a rough plane inclined at 30° to the horizontal.
A force of 80 N acts upward, parallel to the line of greatest slope of the plane.
The box is in limiting equilibrium, on the verge of moving up the plane.



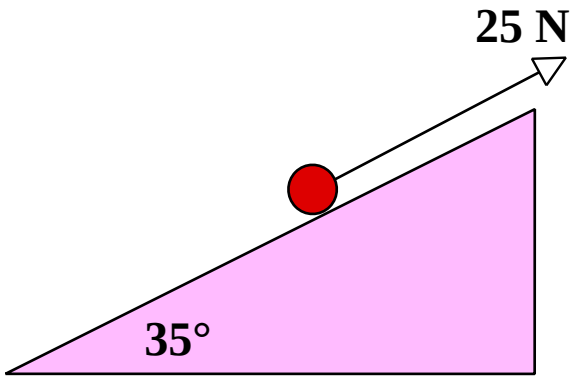
Find **(i)** The friction force, F_r

(ii) The normal reaction, R

(iii) The coefficient of friction, μ

Question 4

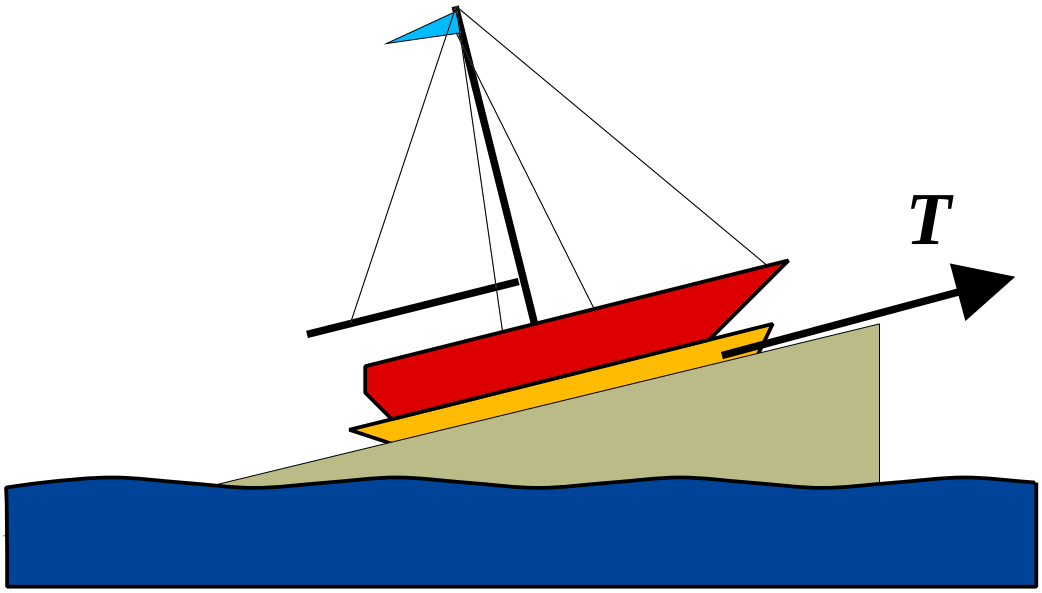
A box of mass 15 kg is at rest on a rough plane inclined at 35° to the horizontal.
A force of 25 N acts upward, parallel to the line of greatest slope of the plane.
The box is in limiting equilibrium, on the verge of moving down the plane.



Find **(i)** The friction force, F_r
(ii) The normal reaction, R
(iii) The coefficient of friction, μ

Question 5

A yacht and cradle of mass 450 kg is held at rest on a slipway by a rope.
The yacht and cradle are modelled as a particle and the slipway considered to be a rough plane inclined at 15° to the horizontal.
The coefficient of friction between the particle and the plane is 0.2
The rope is modelled as a light, inextensible string.
The rope is considered to be parallel to a line of greatest slope of the plane.
The system is at rest but with the particle (yacht and cradle) in limiting equilibrium, on the verge of sliding down the slipway.



(i) Draw a clear diagram showing all significant forces.

(ii) Calculate the tension, T, in the rope.