Lesson 3

A-Level Applied Mathematics : Statistics : Year 2

Correlation II

3.1 Non-Linear Correlation

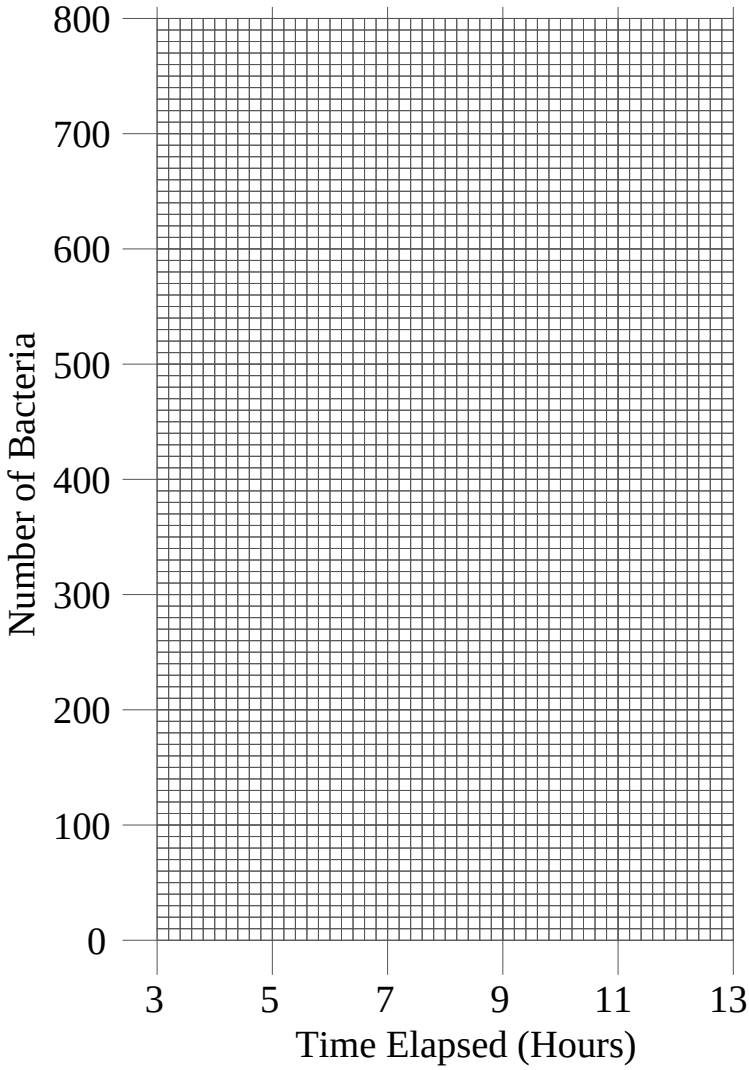
To date when correlation has been found it's been modelled with a ***line*** of best fit. However, sometimes a ***curve*** of best fit is more appropriate.

3.2 Example

The table shows the number a bacteria, y, in a petri-dish after x hours have elapsed.

x hours	3	5	6	8	9	11	12	13
y bacteria	105	150	180	260	310	450	580	740

(i) Plot a scatter graph of the data on the graph paper below;



[4 marks]

(ii) Use your statistics calculator to obtain the Product Moment Correlation Coefficient and the equation of the linear regression line.

[2 marks]

(iii) Add the linear regression line to your part (i) scatter graph.

[2 marks]

(iv) Natalie has read in her biology textbook that bacteria, when unconstrained by food or space, exhibit exponential growth.
She reads that to test for exponential correlation the data is first coded using;

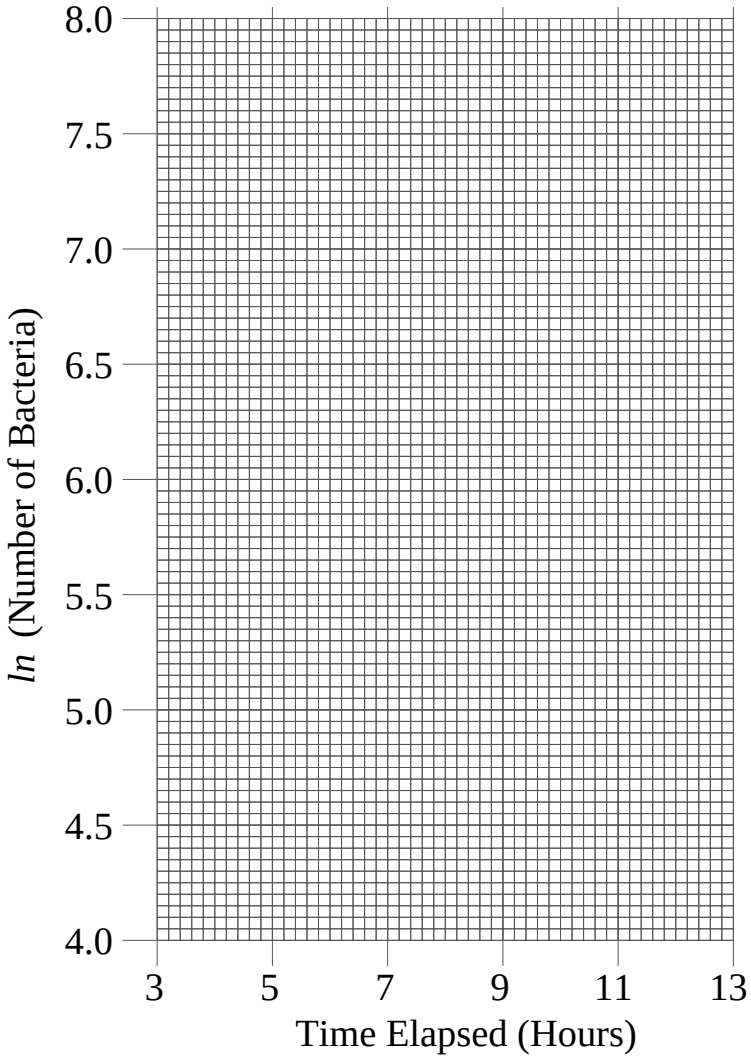
$$Y = ln\, y \qquad \text{and} \qquad X = x$$

and then a plot of Y against X is made.
If a straight line relationship is then evident, the original (uncoded) growth data is indeed better modelled by an exponential curve.
Use this coding to complete the following table;

X	3			8				
$Y = ln\, y$	4.65			5.56				

[2 marks]

(v) Plot a scatter graph of the coded bivariate data below;



[4 marks]

(vi) Is straight line correlation evident in the coded data scatter graph ?

[1 mark]

(vii) Use your statistics calculator to obtain the Product Moment Correlation Coefficient and the equation of the coded data linear regression line.
Add the line to your part (v) scatter graph for the coded data.

[4 marks]

(viii) How does your part (vii) answer confirm that an exponential curve is indeed a better fit to the original (uncoded) data than a straight line ?

[1 mark]

(ix) The general equation of an exponential curve is

$$y = a\,b^x$$

And the equation of the coded data straight line is

$$Y = A + BX$$

The values of a and b are found using the decoding

$$a = e^A \quad \text{and} \quad b = e^B$$

Write out the equation of the exponential curve of best fit in the form

$$y = a\,b^x$$

where a and b are numbers, correct to three significant figures that you have determined.

[3 marks]

(x) Carefully plot the exponential curve of best fit onto your part (i) graph.

[2 marks]

(xi) There is a way of getting the equation of the exponential curve of best fit from your statistics calculator directly from the original uncoded table of data.
See if you can puzzle out how to do this.

[1 mark]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School
It may be freely duplicated and distributed, unaltered, for non-profit educational use
In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"
© 2021 Number Wonder
Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk