Lesson 2

A-Level Pure : The Binomial Theorem : Year 2

2.1 Pascals' Triangle

The first few entries in Pascal's Triangle are;

1									
1	1								
1	2	1							
1	3	3	1						
1	4	6	4	1					
1	5	10	10	5	1				
1	6	15	20	15	6	1			
1	7	21	35	35	21	7	1		
1	8	28	56	70	56	**28**	8	1	
1	9	36	84	126	126	84	36	9	1

In general, entries in the triangle are referred to by describing which row and column they are in. However, there are a couple of catches in addition to the columns being askew.

The first row is termed Row 0, and the letter n is used for the Row number.
The first column is termed Column 0, and the letter r is used for the Column number.

The description can be written in two forms;

$$\binom{n}{r} = {}^{n}C_{r}$$

So, the highlighted cell which contains the number 28 is in Row 8, Column 6.

Check that you can extract the 28 from your calculator,

$$\binom{8}{6} = {}^{8}C_{6} = 28$$

There is also a formula for calculating the entries in the triangle,

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

The factorials generate big numbers and you may find that your calculator can't handle the numbers for even modest values of n
70! is the first factorial that my Casio fx-991 EX Classwiz can't handle.

2.2 Example

Find an expression in terms of *a* that contains no factorials for

$$^{a+6}C_{a+4}$$

where *a* is an unknown integer constant.

2.3 Exercise

Question 1

Use your calculator to work out;

(i)

$$7\,!$$

(ii)

$$0\,!$$

Question 2

Without using a calculator, determine the exact value of

(i)

$$\frac{100\,!}{98\,!}$$

(ii)

$$\frac{100\,!}{2\,!\;(100-2)\,!}$$

Question 3

Find an expression in terms of *a* that contains no factorials for

$$^{a+11}C_{a+9}$$

where *a* is an unknown integer constant.

Question 4
Use your calculator to determine,

(i)

$$\binom{11}{7}$$

(ii)

$$^{14}C_5$$

Question 5
From our Year 1 course we have that

$$\begin{aligned}(a + bx)^n = &\binom{n}{0} \times a^n \times (bx)^0 \\ &+ \binom{n}{1} \times a^{n-1} \times (bx)^1 \\ &+ \binom{n}{2} \times a^{n-2} \times (bx)^2 \\ &+ \binom{n}{3} \times a^{n-3} \times (bx)^3 \\ &+ \dots\end{aligned}$$

We are now going to assume $a = b = 1$ in the above and replace the Pascal Triangle coefficients using

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Thus, determine the first four terms of The Binomial Expansion of $(1 + x)^n$

$$(1 + x)^n = 1 + nx +$$

Question 6

The Binomial Theorem states that...

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

... provided $|x| < 1$

and, amazingly, this version is valid for n being any integer, even a negative.

Use the binomial theorem to find the first four terms in the expansion of...

$$\frac{1}{(1 + 6x)}$$

... and state the values of x for which your expansion is valid.

Question 7

The Binomial Theorem states that...

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

... provided $|x| < 1$

and, incredibly, this version is is valid for all rational numbers.

Use the binomial theorem to find the first four terms in the expansion of...

$$\sqrt{1 + 3x}$$

... and state the values of x for which your expansion is valid.

Question 8

The Binomial Theorem states that...

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

... provided $|x| < 1$

Use the binomial theorem to find the first four terms in the expansion of...

$$\frac{1}{(1-x)^2}$$

... and state the values of x for which your expansion is valid.

Question 9

The Binomial Theorem states that...

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

... provided $|x| < 1$

Use the binomial theorem to find the first four terms in the expansion of...

$$\frac{1}{(3 + x)}$$

... and state the values of x for which your expansion is valid.