

## Lesson 3

### A-Level Pure : The Binomial Theorem : Year 2

#### 3.1 The Binomial Theorem & Infinite Series Expansion

The Binomial Theorem provides a powerful means of expanding the brackets of expressions such as;

$$\frac{1}{(1+x)} \quad (1+7x)^{0.5} \quad (2+3x)^{-4} \quad \frac{6}{\sqrt{5x+6}}$$

The expansion is typically into an infinite series which will converge provided  $x$  is sufficiently small.

#### 3.2 The Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

The number 1, prior to bracket expansion, is crucial.

The series converges provided  $|x| < 1$ .

#### 3.3 Worked Example

*C4 examination question from January 2013, Q1*

Given

$$f(x) = (2+3x)^{-3} \quad |x| < \frac{2}{3}$$

find the binomial expansion of  $f(x)$ , in ascending powers of  $x$ , up to and including the term in  $x^3$ . Give each coefficient as a simplified fraction.

[ 5 marks ]

### 3.4 Exercise

#### Question 1

*C4 examination question from June 2007, Q1*

$$f(x) = (3 + 2x)^{-3} \quad |x| < \frac{3}{2}$$

Find the binomial expansion of  $f(x)$ , in ascending powers of  $x$ , as far as the term in  $x^3$   
Give each coefficient as a simplified fraction.

[ 5 marks ]

**Question 2**

*C4 examination question from January 2008, Q2*

(a) Use the binomial theorem to expand

$$(8 - 3x)^{\frac{1}{3}} \quad |x| < \frac{8}{3}$$

in ascending powers of  $x$ , up to and including the term in  $x^3$   
Give each term as a simplified fraction.

[ 5 marks ]

(b) Use your expansion, with a suitable value of  $x$ , to obtain an approximation to  $\sqrt[3]{7.7}$ . Give your answer to 7 decimal places.

[ 2 marks ]

**Question 3**

*C4 examination question from January 2012, Q3*

(a) Expand

$$\frac{1}{(2 - 5x)^2} \quad |x| < \frac{2}{5}$$

in ascending powers of  $x$ , up to and including the term in  $x^2$   
Give each term as a simplified fraction.

[ 5 marks ]

Given that the binomial expansion of

$$\frac{2 + kx}{(2 - 5x)^2}, \quad |x| < \frac{2}{5}$$

is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

(b) find the value of the constant  $k$

[ 2 marks ]

(c) find the value of the constant  $A$

[ 2 marks ]

**Question 4**

*C4 examination question from June 2008, Q5*

(a) Expand

$$\frac{1}{\sqrt{4-3x}} \quad |x| < \frac{4}{3}$$

in ascending powers of  $x$ , up to and including the term in  $x^2$   
Simplify each term.

[ 5 marks ]

(b) Hence, or otherwise, find the first 3 terms in the expansion of

$$\frac{x+8}{\sqrt{4-3x}}$$

as a series in ascending powers of  $x$

[ 4 marks ]

All examination questions are © Pearson Education Ltd  
and have appeared in the Edexcel GCE (A level) Pure Mathematics examination papers

These lesson notes are available from [www.NumberWonder.co.uk](http://www.NumberWonder.co.uk)  
They may be freely duplicated and distributed but copyright remains with the author.

© 2019 Number wonder