

Lesson 2

A-Level Applied Mathematics : Statistics : Year 2 Set Theory and Probability II

2.1 Conditional Probability

Considering the probabilities associated with a mathematics test.

Either a candidate revises or they don't.

Either a candidate passes the test or they don't.

Three pieces of information are given;

- ◇ The probability that a student revises for a test is 0.9
That is, $P(\text{revise}) = 0.9$
 - ◇ If the student revises then the probability of passing the test is 0.8
That is, $P(\text{pass} | \text{revise}) = 0.8$
 - ◇ If the student does not revise then the probability of passing is reduced to 0.4
That is, $P(\text{pass} | \text{revise}') = 0.4$
- (i) Draw a tree diagram to show this information.

[3 marks]

- (ii) What is the probability that a student picked at random has not revised, but has passed the test ?

[1 mark]

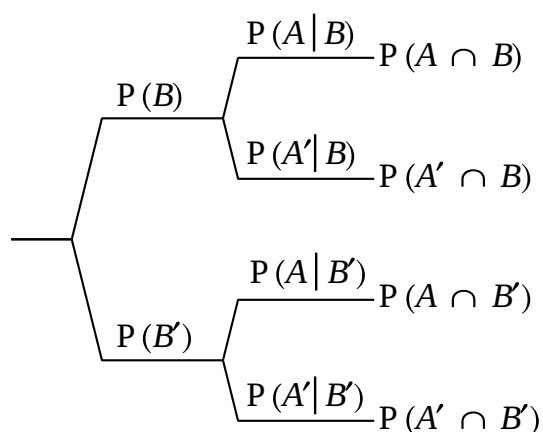
2.2 It's A Given

Given That

The probability of event A happening given that event B has already happened is written as $P(A|B)$

2.3 A Generalised Tree Diagram

Here is a generalised tree diagram for two events, A and B with event B occurring first.



$$P(B) \times P(A|B) = P(A \cap B)$$

$$\Leftrightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This “given that” formula needs to be memorised.

2.4 Reformulation

Write down similar formulae for;

(i) $P(B'|A) =$

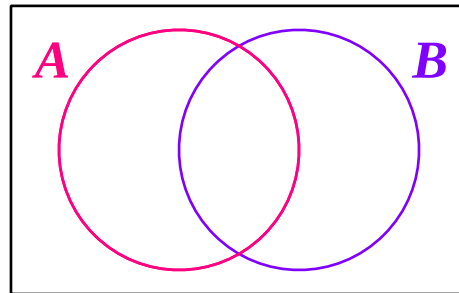
(ii) $P(E|Z) =$

[1, 1 marks]

2.5 Example

Given that, $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$

(a) Placing appropriate probabilities in the regions on the Venn diagram.



[2 marks]

(b) State the values of the following probabilities;

(i) $P(A \cup B)$

[1 mark]

(ii) $P(A|B)$

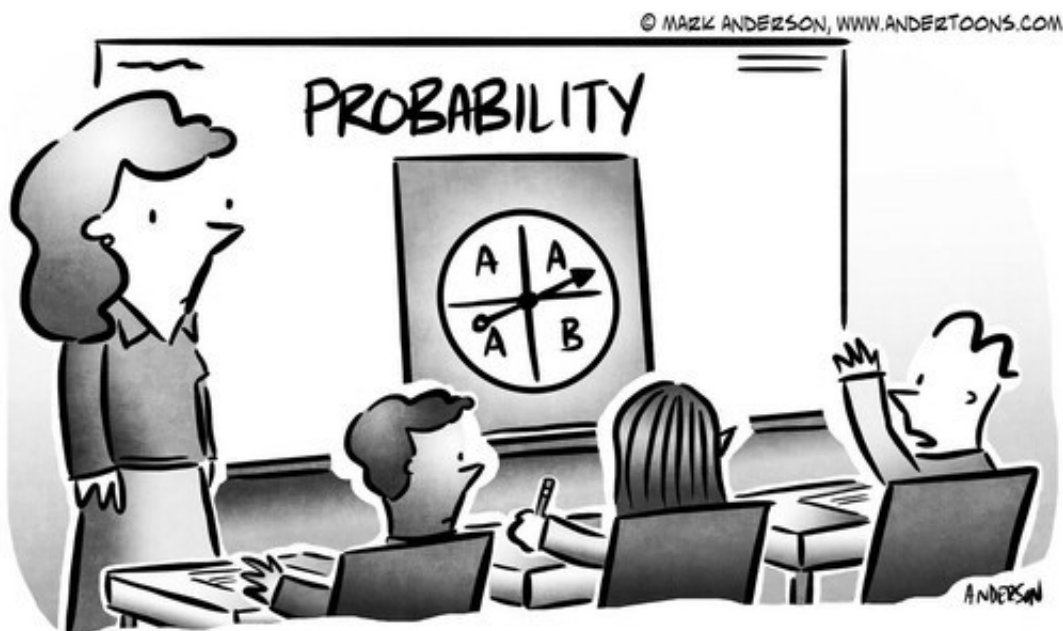
[1 mark]

(iii) $P(A')$

[1 mark]

(iv) $P(B|A')$

[1 mark]



"I know mathematically that A is more likely,
but I gotta say, I feel like B wants it more."

2.6 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 55

Question 1

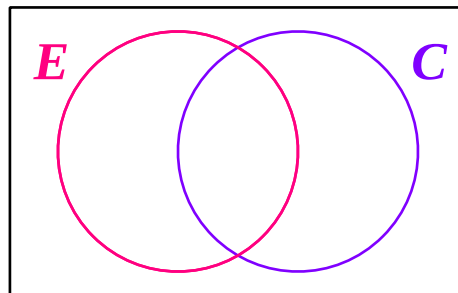
A school has 140 students in its upper sixth.

85 study English.

24 study English and Chemistry.

82 don't study Chemistry.

Draw a Venn Diagram to show this information.



[2 marks]

An upper sixth student from the school is selected at random.

What is;

(i) $P (C)$

(ii) $P (E ')$

(iii) $P (E \cap C ')$

(iv) $P (E \cup C)$

(v) $P (E ' \cap C ')$

(vi) $P (E | C)$

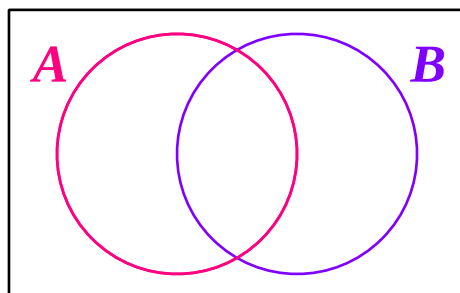
(vii) $P (C | E)$

(viii) $P (E ' | C)$

[8 marks]

Question 2

- (a) Given that, $P(A) = 0.7$, $P(B) = 0.5$ and $P(A \cap B) = 0.45$, complete the Venn Diagram to show the appropriate probabilities.



[2 marks]

- (b) What is;

(i) $P(A')$

(ii) $P(A \cup B)$

(iii) $P(A \cap B')$

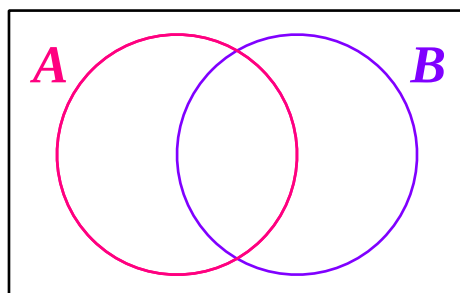
(iv) $P(A|B)$

[4 marks]

Question 3

Recall that, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

- (a) Given that, $P(A) = 0.7$, $P(B) = 0.5$ and $P(A|B) = 0.6$ complete the Venn Diagram to show the appropriate probabilities.



[2 marks]

- (b) What is;

(i) $P(A \cap B)$

(ii) $P(B')$

(iii) $P(A \cup B)'$

(iv) $P(B|A)$

[4 marks]

Question 4

The probability that a student revises for a test is 0.8

If the student revises then the probability of passing the test is 0.7

If the student does not revise then the probability of passing is reduced to 0.3

- (i) Draw a tree diagram to show this information.
- (ii) Calculate the probability that a randomly chosen student passes the test.
- (iii) Janet† passed the test. Find the probability she revised.
- (iv) Rosa† passed the test. Find the probability she did not revise.
- (v) Bubbles† did not pass the test. Find the probability that she revised.

[10 marks]

† Any similarities between these names and students you may know is a coincidence.
It is well known that all students in Mr Hansen's class will revise and will pass the test.

Question 5

A market researcher asked 100 adults which of three newspapers they read.
The results showed that;

- ◇ 30 read The Guardian
- ◇ 26 read The Times
- ◇ 21 read The Independent
- ◇ 5 read both The Guardian and The Times
- ◇ 7 read both The Times and the Independent
- ◇ 6 read both The Independent and The Guardian
- ◇ 2 read all three.

(i) Draw a Venn diagram to represent these data.

[6 marks]

One of the adults is then selected at random.

Find the probability that she reads

(ii) at least one of the newspapers

[2 marks]

(iii) only The Guardian

[1 mark]

(iv) only one of the newspapers

[2 marks]

(v) The Guardian given that she reads only one newspaper

[2 marks]

Question 6

In a particular school 55% of the pupils are girls.

During a games afternoon 65% of the pupils play tennis.

Only 40% of the boys play tennis.

A pupil is selected at random.

Find the probability that;

- (i) The pupil is a girl who plays tennis
- (ii) The pupil is a boy who does not play tennis.
- (iii) The pupil is a girl *given that* the pupil plays tennis.

[10 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk