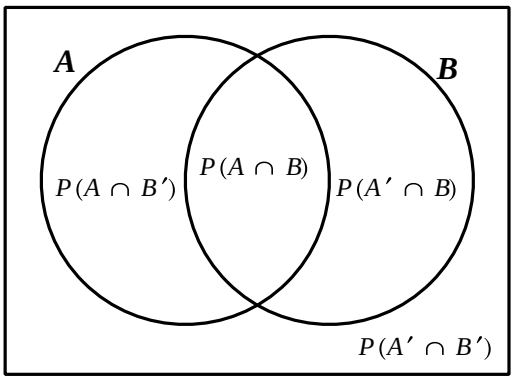
3.1 Contingency Tables

(Also called **Two-Way Tables** and **Crosstabs**)

Here is a generalised two-hoop Venn Diagram;



Another useful problem solving tool is a Contingency Table, often called a Two-Way Table, or a Crosstabs;

	A	***A'***	***Sum***
B	$P(A \cap B)$	$P(A' \cap B)$	$p(B)$
B'	$P(A \cap B')$	$P(A' \cap B')$	$p(B')$
Sum	$p(A)$	$p(A')$	1

A couple of small advantages of such tables, in comparison with a Venn diagram are that the numbers are to hand to calculate "given that" probabilities;

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

and also those required to test for independence;

$$\textit{Independence} \Leftrightarrow P(A \cap B) = P(A) \times P(B)$$

3.2 Example

Two fair dice, one red and one green, are rolled.
Event *D* is the occurrence of the same number rolled on each dice; a "double".
Event *N* is the occurrence of a sum of 9 or more.

(i) Complete the two-way table with the appropriate probabilities.

	D	***D'***	***Sum***
N			
N'			
Sum			

[3 marks]

(ii) Show with the appropriate test that events *D* and *N* are not independent.

[2 marks]

(iii) Find the following "given that" probability;

$$P(D \mid N)$$

[2 marks]

3.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available: 50

Question 1

The fans of a football club are asked if they think the manager should resign or stay. The results are presented in the two-way table below;

	Resign	***Stay***	***Sum***
Male	**0.6**	**0.2**	**0.8**
Female	**0.1**	**0.1**	**0.2**
Sum	**0.7**	**0.3**	**1.0**

(a) If a fan is picked at random, find

(i) P (*Male*)

[1 mark]

(ii) the probability they are a male who thought the manager should stay

[1 mark]

(iii) P (*Female* | *Resign*)

[2 marks]

(b) What is the probability that a male fan, picked at random, thinks the manager should resign ?

[2 marks]

(c) Amongst the fans, is thinking the manager should resign independent of being male ? Give a reason for your answer.

[2 marks]

Question 2

(i) Complete the following two-way table

	A	A'	*Sum*
B		0.4	
B'	0.3		
Sum	0.5		1.0

[2 marks]

(ii) Are events A and B independent ?
Give a calculation to support your answer.

[2 marks]

Question 3

The *Bog-Trotters* are a rambling group is made up of 63 women and 37 men.
A walking stick is used by 26 of the women and 18 of the men.

(a) Draw a two-way table to show this information.

[4 marks]

(b) One rambler from the *Bog-Trotters* is chosen at random.
Find;

(i) P (*Uses a stick*)

[1 mark]

(ii) P (*Uses a stick* | *Female*)

[1 mark]

(iii) P (*Male* | *Uses a stick*)

[1 mark]

Question 4

Two fair dice, one red and one yellow, are rolled.
Event *E* is the occurrence of an even number rolled on each dice; an "even snap".
Event *R* is the occurrence of a higher score on the red dice than on the yellow.

(i) Complete the two-way table with the appropriate probabilities.

	E	***E'***	***Sum***
R			
R'			
Sum			

[3 marks]

(ii) Show with the appropriate test that events *E* and *R* are not independent.

[2 marks]

(iii) Find the following "given that" probability;

$$P(E \mid R)$$

[2 marks]

Question 5

Complete the contingency table of probabilities given that events W and Z are statistically independent.

	W	W'	Sum
Z			0.25
Z'			
Sum	0.2		

[3 marks]

Question 6

(i) Complete the contingency table of probabilities.

	T	T'	Sum
V			0.4
V'		0.5	
Sum	0.3		

[2 marks]

(ii) Are events T and V independent ?
Give a calculation to support your answer.

[2 marks]

(iii) Are events T and V mutually exclusive ?
Give a calculation to support your answer.

[2 marks]

Question 7

Two fair dice, one red and one green, are rolled.
Two events are defined as follows;
E : The sum of the two scores is Eight or more
L : At least one dice has rolled a score of One

(i) Complete the two-way table with the appropriate probabilities.

	E	***E'***	***Sum***
L			
L'			
Sum			

[2 marks]

(ii) Are events E and L independent ?
Give a reason for your answer.

[2 marks]

(iii) Are events E and L mutually exclusive ?
Give a reason for your answer.

[2 marks]

(iv) Find the following "given that" probability;

$$P\,(L' \mid E)$$

[2 marks]

Question 8

Anna and Bella are sometimes late for school.
The events A and B are defined as follows;
A is the event that Anna is late for school.
B is the event that Bella is late for school.

$\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(A' \cap B') = 0.1$

On a randomly selected day, find the probability that;

(i) both Anna and Bella are late to school

[2 marks]

(ii) Anna is late to school given that Bella is late to school

[2 marks]

Their teacher suspects that Anna and Bella being late for school is linked in some way.

(iii) Comment on his suspicion, showing your working.

[3 marks]