Set Theory and Probability II

#### 4.1 The Addition Rule

At GCSE level, the phrase is often used that "or means add" and this is true when two events, *A* and *B*, are mutually exclusive.

That is 
$$P(A \cup B) = P(A) + P(B)$$
 when  $P(A \cap B) = \emptyset$ 

However, the full rule, without the "mutually exclusive" restriction is this;

#### **The Addition Rule**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

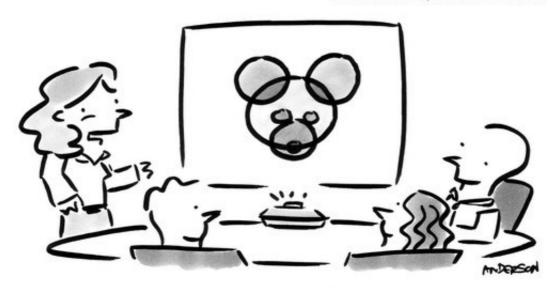
Often the rule is rearranged and used to find  $P(A \cap B)$ 

#### Proof

The truth of the Addition Rule is made obvious by thinking of Venn Diagrams;

$$\begin{bmatrix} A & B \\ B & B \end{bmatrix} = \begin{bmatrix} A & B \\ B & B \end{bmatrix} + \begin{bmatrix} A & B \\ B & B \end{bmatrix} - \begin{bmatrix} A & B \\ B & B \end{bmatrix}$$

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"This Venn diagram tells us nothing, but it's so cute!"

#### 4.2 Exercise

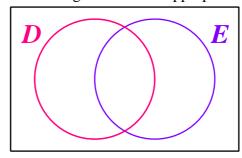
Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 50

### **Question 1**

Given that P(D) = 0.6, P(E) = 0.3 and  $P(D \cup E) = 0.8$ ,

(a) annotate the Venn Diagram with the appropriate probabilities.



[ 3 marks ]

(**b**) Find,

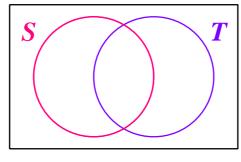
- (i)  $P(D \cap E)$
- (ii)  $P(D' \cap E)$
- (iii)  $P(D \cap E')$
- (iv)  $P(D' \cap E')$
- $(\mathbf{v}) \quad P(D \cup E')$
- (vi)  $P(D' \cup E)$

[6 marks]

### **Question 2**

Given that, P(T) = 0.4,  $P(S \cap T) = 0.15$  and  $P(S' \cap T') = 0.5$ .

(a) annotate the Venn Diagram with the appropriate probabilities.



[ 3 marks ]

(**b**) Find,

- (i)  $P(S \cap T')$
- (ii) P(S)
- (iii)  $P(S \cup T)$
- (iv)  $P(S' \cap T)$
- $(\mathbf{v}) \quad P(S' \cup T')$

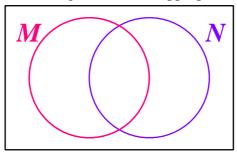
[ **5** marks ]

# **Question 3**

The events M and N are such that  $P(M) = P(N) = 2P(M \cap N)$ 

Furthermore,  $P(M \cup N) = 0.6$ 

(a) annotate the Venn Diagram with the appropriate probabilities.



[3 marks]

- (**b**) Find,
  - (i)  $P(M \cap N)$
- (ii) P(M)
- (iii)  $P(M' \cap N')$
- (iv)  $P(M \cap N')$

[4 marks]

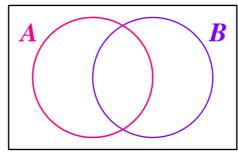
# **Question 4**

Of the households in Shrewsbury, 85% have an internet connection and 35% have satellite TV. Given that 25% of the households have both an internet connection and satellite TV, calculate the percentage probability that a household has either an internet connection or satellite TV but not both.

# **Question 5**

Given that, P(A) = 0.7, P(B) = 0.4 and  $P(A \cup B) = 0.8$ ,

(a) annotate the Venn Diagram with the appropriate probabilities.



[ 3 marks ]

- (b) Find,
  - (i)  $P(A \cap B)$
- (ii)  $P(A \mid B)$

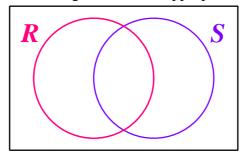
- (iii)  $P(B \mid A)$
- (iv)  $P(A \mid B')$

[4 marks]

# **Question 6**

Given that,  $P(R \mid S) = 0.5$ ,  $P(R \mid S')$  and P(S) = 0.6,

annotate the Venn Diagram with the appropriate probabilities. (a)



[ 3 marks ]

- (b) Find,
  - (i) P(R)

( ii )  $P(S \mid R)$ 

- (iii) P(S' | R) (iv) P(S' | R')

[4 marks]

# **Question 7**



In a forest are red squirrels and grey squirrels.

55% of the squirrels are red.

During a sunny autumn afternoon 65% of the squirrels gather nuts.

Only 40% of the grey squirrels gather nuts.

A squirrel is selected at random.

Find the probability that the squirrels is,

- (i) a red who is gathering nuts,
- (ii) a grey who is not gathering nuts,
- (iii) a red given that it is gathering nuts.

[ 7 marks ]