

4E.1 The Sum To Infinity Of A Geometric Progression

Remarkably, under certain conditions, an infinite addition of the following form can have an answer that is a fixed and finite value.

$$Sum = a + ar + ar^2 + ar^3 + ar^4 + \dots$$

The individual terms in this sum form what is called a Geometric Progression with first term, a , and common ratio, r

The sum to infinity exists if $-1 < r < 1$ and is given by the formula;

$$Sum_{\infty} = \frac{a}{1 - r}$$

A famous example of such a sum is;

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \dots$$

In this famous example, $a = 1$ and $r = 0.5$

$$\begin{aligned} Sum_{\infty} &= \frac{1}{1 - 0.5} \\ &= 2 \end{aligned}$$

4E.2 Example

Tom and Jerry take turns to spin a fair coin.

The person to spin Heads first wins.

Tom goes first. What is the probability that he wins ?

4E.3 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available: 50

Question 1

Consider each of these infinite addition sums.

Each has terms in Geometric Progression.

In each case state the value of the first term, a , and the value of the common ratio, r .

Then determine the value of the sum using the formula for the sum to infinity.

(i) $A = 15 + 3 + 0.6 + 0.12 + 0.024 + \dots$

[2 marks]

(ii) $B = 4 + 3.2 + 2.56 + 2.048 + 1.6384 + \dots$

[2 marks]

(iii) $C = 8 + \frac{8}{3} + \frac{8}{9} + \frac{8}{27} + \frac{8}{81} + \frac{8}{243} + \dots$

[2 marks]

(iv) $D = 2 + 0.2 + 0.02 + 0.002 + 0.0002 + \dots$

[2 marks]

Question 2

Bill and Ben take turns to spin a coin, biased with probability 0.4 of landing Heads.

The person to spin Heads first wins.

Bill goes first.

- (i) Explain why the probability Bill wins on his second turn is $0.6 \times 0.6 \times 0.4$

[2 marks]

- (ii) Show that the probability Bill wins on his third turn is 0.05184

[2 marks]

- (iii) Using the sum to infinity formula, determine the probability Bill wins.

[3 marks]

Question 3

Lucy and Billy take turns to spin a coin, biased with probability 0.3 of landing Heads.

The person to spin Heads first wins.

Lucy goes first.

What is the probability that Lucy wins ?

[4 marks]

Question 4

Sid and Vicky are discussing whether to go to the cinema or to bingo.



They decide to take it in turns to roll a six sided die.
Who rolls a six first gets to decide where they go.

Sid goes first.

(i) What is the probability that Sid will be the first to roll a 6 ?

[5 marks]

(ii) Is this method of resolving a dispute fair ?
Give a reason for your answer.

[2 marks]

Question 5

After another argument, Sid and Vicky decide to use a twelve sided die.



Sid again goes first.

(i) What is the probability that Sid will be the first to roll a 12 ?

[5 marks]

(ii) Is this method of resolving a dispute more or less fair than using a six sided die ?

[1 mark]

(iii) Is there another platonic solid that would make a more fair die ?

[1 mark]

Question 6

Sam and Joe take turns to spin a coin, biased with probability β of landing Heads.
The person to spin Heads first wins.

Sam goes first.

(i) What is the probability in terms of β that Sam wins ?

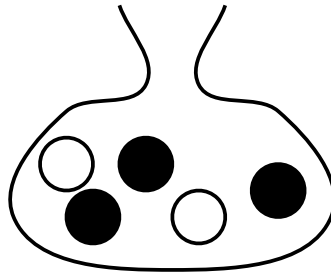
[6 marks]

Try to find a value of β which ensures that Sam and Joe have equal chances
in the game, if Sam goes first.

[3 marks]

Question 7

Two players, Fred and Harry, take turns to draw a ball from a bag and replace it.
The bag contains two white and three black balls.



The first player, Fred, has to draw a white ball to win.
The second player, Harry, has to draw a black ball to win.
(i) Find the probability that Fred will win.

[7 marks]

(ii) Which player has the better chance of winning ?

[1 mark]