Lesson 5

A-Level Applied Mathematics : Statistics : Year 2
Set Theory and Probability II

5.1 From The Examination Hall

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available: 66

Question 1
Let A and B be events such that;

- $\mathrm{P}(A) = \frac{1}{4}$
- $\mathrm{P}(B) = \frac{1}{3}$
- $\mathrm{P}(A \cup B) = \frac{5}{12}$

(i) Show the relationship between A and B on a Venn Diagram.

[3 marks]

(ii) Find $\mathrm{P}(A \mid B)$

[2 marks]

Question 2

There are 60 students in the sixth form of a certain school.
Mathematics is studied by 27 of them, biology by 20 and 22 students study neither mathematics nor biology.

(i) Show how you would use the probability addition formula,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

to find the probability that a randomly selected student studies both mathematics and biology.

[3 marks]

(ii) Find the probability that a randomly selected mathematics student does not study biology.

[3 marks]

A student is selected at random.

(iii) Determine whether the event “studying mathematics” is statistically independent of the event “not studying biology”.

[4 marks]

Question 3

A school allows students 3 attempts to pass a Maths test
before EXPELLING the student.

The probability that a student passes first time is 0.4
On the student's second attempt, the probability of passing is 0.7
On the third attempt, the probability of passing is 0.2

(i) Draw a tree diagram to show this information.

[4 marks]

(ii) Find the probability that a randomly selected student will pass the test.

[2 marks]

(iii) Mia passed the test.
Find the probability that she passed first time.

[2 marks]

(iv) If, before the test, the school had 200 mathematicians, how many is it likely to have after the test ?

[2 marks]

Question 4

A group of 100 people produced the following information relating to three attributes.
The attributes were;

- ◇ wearing glasses
- ◇ being left handed
- ◇ having dark hair

Glasses were worn by 36 people, 28 were left handed and 36 had dark hair.
There were 17 who wore glasses and were left handed.
There were 19 who wore glasses and had dark hair.
There were 15 who were left handed and had dark hair.
Only 10 people wore glasses, were left handed and had dark hair.

(a) Represent these data on a Venn diagram.

[6 marks]

A person was selected at random from this group.
Find the probability that this person;

(b) wore glasses but was not left handed and did not have dark hair.

[1 mark]

(c) did not wear glasses, was not left handed and did not have dark hair.

[1 mark]

(d) had only two of the attributes.

[2 marks]

(e) wore glasses given that they were left handed and had dark hair.

[3 marks]

Question 5

In the 6th Form at a particular school;

- ◇ $\frac{1}{2}$ of the pupils study Maths
- ◇ $\frac{1}{4}$ of the pupils study Physics
- ◇ $\frac{7}{16}$ of the pupils study neither Maths nor Physics

(i) Draw a Venn diagram to show this information.

[4 marks]

(ii) What is the probability that a randomly chosen student studies Maths and Physics ?

[1 mark]

(iii) Fiona studies Physics.
What is the probability that she also studies Maths ?

[2 marks]

(iv) Frank does not study maths.
What is the probability that he studies Physics ?

[2 marks]

The 6th Form contains 240 pupils.
I stop two of the pupils at random.

(v) Calculate the probability that

(a) they both study Maths.

[2 marks]

(b) one studies Maths and the other does not.

[2 marks]

Question 6

Given that;

$$\mathrm{P}(A) = \frac{1}{2} \qquad \mathrm{P}(B) = \frac{1}{3} \qquad \mathrm{P}(B \mid A) = \frac{2}{3}$$

Find **(i)** $\mathrm{P}(B' \mid A)$

[3 marks]

(ii) $\mathrm{P}(A \mid B')$

[3 marks]

Question 7

(a) State in words the relationship between two events R and S when

$$\mathrm{P}(R \cap S) = 0$$

[1 mark]

The events A and B are independent with

$$\mathrm{P}(A) = \frac{1}{4} \quad \text{and} \quad \mathrm{P}(A \cup B) = \frac{2}{3}$$

Find

(b) $\mathrm{P}(B)$

[4 marks]

(c) $\mathrm{P}(A' \cap B)$

[2 marks]

(d) $\mathrm{P}(B' \mid A)$

[2 marks]

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