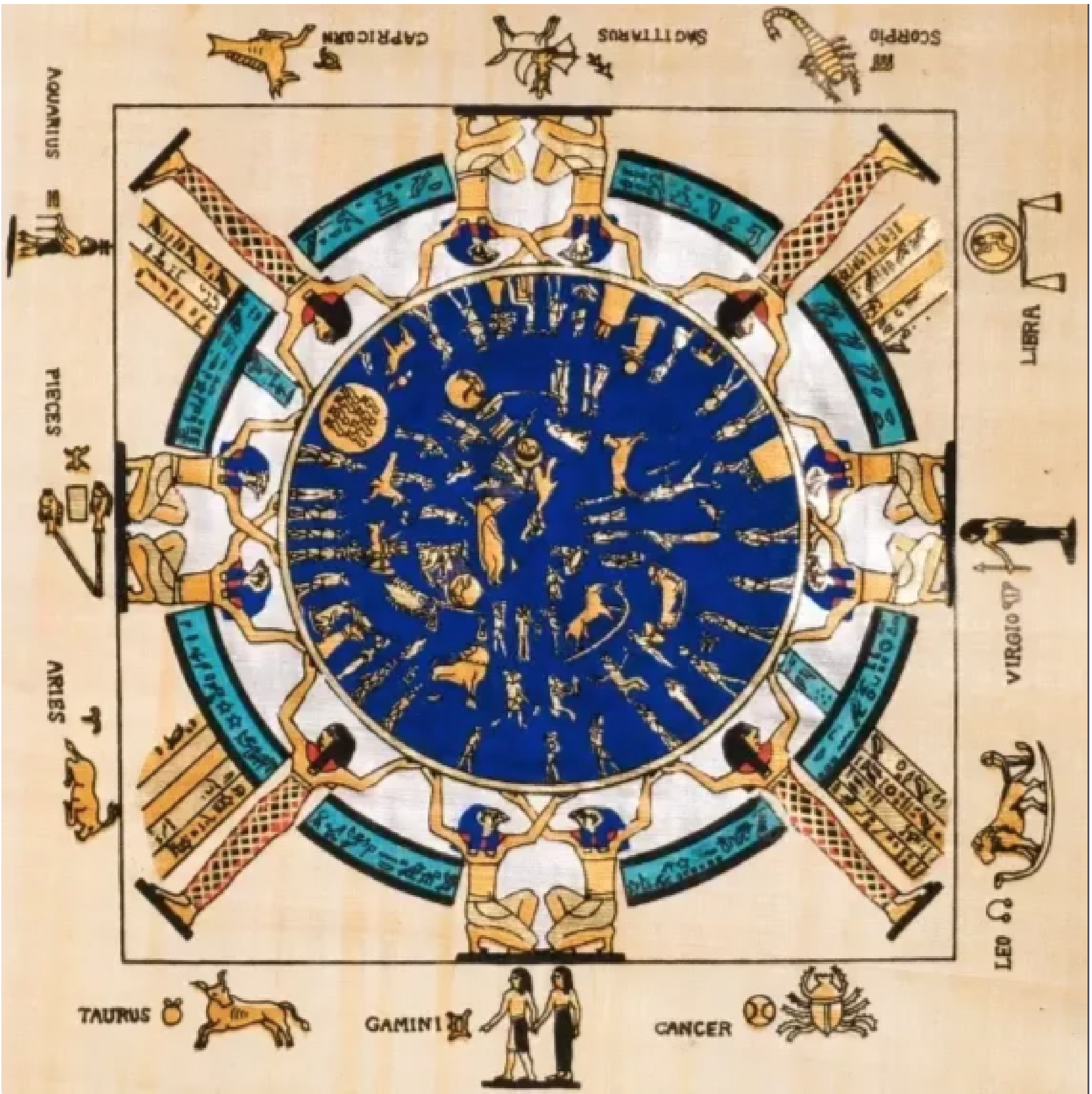
A-Level Pure Mathematics
Year 2

Applications of Trigonometry



On ancient Egyptian calendars, there were 360 days in the year.
Five days of festivities followed the end of each calendar year before the start of the next.
Possibly, this is the origin of there being 360° in a circle

Lesson 1

1.1 Measuring Angles In Degrees

Ancient Persian (Iranian) and Egyptian calendars had 12 months of 30 days in a year. It's speculated that this is how our division of a circle into 360 degrees arose, with each day allocated 1 degree. The Persians and Egyptians appreciated the mathematical properties of 360, particularly its abundance of factors.

The 24 Factors of 360 :
1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360

Every six years, the Persians added a 13th month to keep their calendar synchronized with the seasons, whereas the Egyptians had five additional days of festivities between the end of one year and the start of the next.

1.2 Measuring Angles In Radians

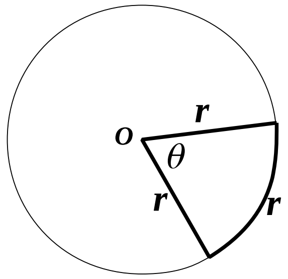
From the mathematician's point of view, it turns out that to divide a circle into 360 parts is not the most natural. Instead, it makes more mathematical sense to break it into 2π parts. These mathematical parts are called radians.

$$\textit{one radian} \quad = \quad 57.29577951\ldots \textit{ degrees}$$

Many mathematical properties, formulae and theorems are far more elegantly presented when the angles involved are expressed in radians, rather than degrees. Some of these will be encountered soon.

1.3 The Definition of a Radian

One radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.



Angle θ is 1 radian

1.4 Two Elegant Formulae

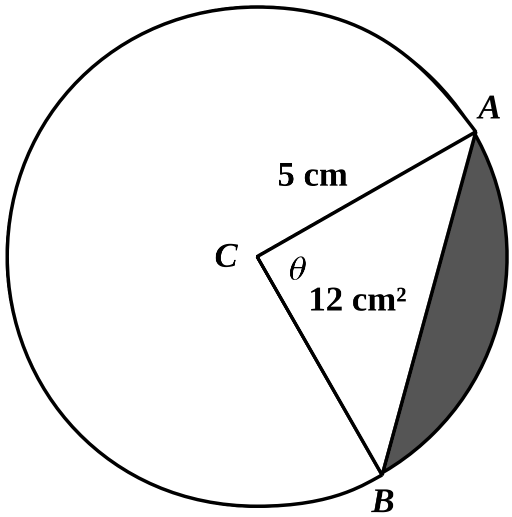
Arc Length and Sector Area

$$\textit{Arc length} \quad = \quad r\,\theta^c$$

$$\textit{Sector area} \quad = \quad \frac{1}{2}\,r^2\,\theta^c$$

1.5 Example

The circle shown, centre C, has radius 5 cm



The area of $\triangle ABC$ is 12 cm^2. Find, to three decimal places,

(i) the angle marked θ, in radians,

(ii) the area of the segment shaded,

(iii) the perimeter of the shaded segment.

Teaching Video : http://www.NumberWonder.co.uk/v9057/1a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9057/1b.mp4 (Part 2)

<== Part 1 Part 2 ==>

After watching the teaching video write out a solution to the question.

☞

[7 marks]

1.6 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

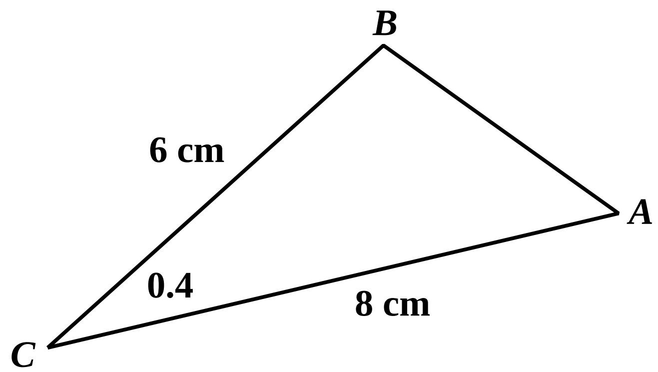
Put your calculator into RADIAN mode

Find, accurate to four decimal places, the value of,

(i) $\sin 1.5^c$ **(ii)** $\cos 0.8^c$

[2 marks]

Question 2



(i) Determine the area of $\triangle ABC$, to 2 decimal places, by using the formula,

$$Area\,\triangle \;=\; \frac{1}{2}\,a\,b\,\sin C^{c}$$

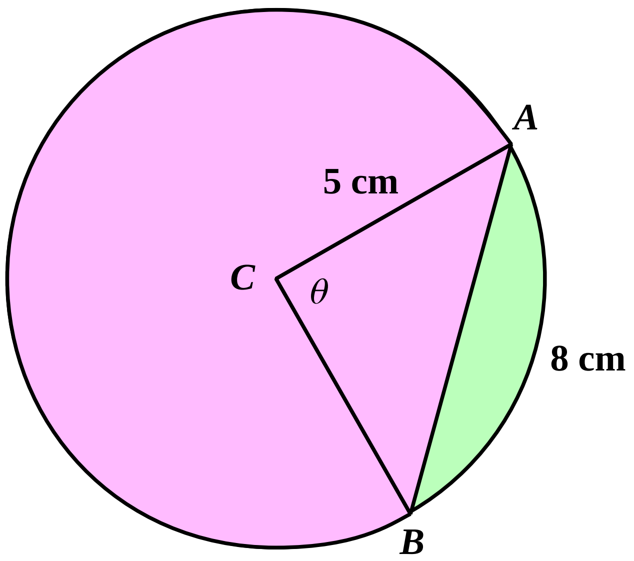
[2 marks]

(ii) Find the length of AB, to 2 decimal places, by using the formula,

$$c^2 \;=\; a^2 + b^2 - 2\,a\,b\cos C^{c}$$

[2 marks]

Question 3



(i) Find, in radians, the angle marked θ

[2 marks]

(ii) Find the area of the sector *ABC*

[2 marks]

(iii) Find, to 2 decimal places, the area of $\triangle ABC$

[2 marks]

(iv) Find the area of the minor segment, shaded green

[2 mark]

(v) Find the area of the major segment, shaded purple

[2 marks]

Question 4

Put your calculator into RADIAN mode.

Work out the acute angle, in radians and correct to three significant figures, of,

(i) *arccos* 0.9 **(ii)** *arcsin* 0.92

[2 marks]

Question 5

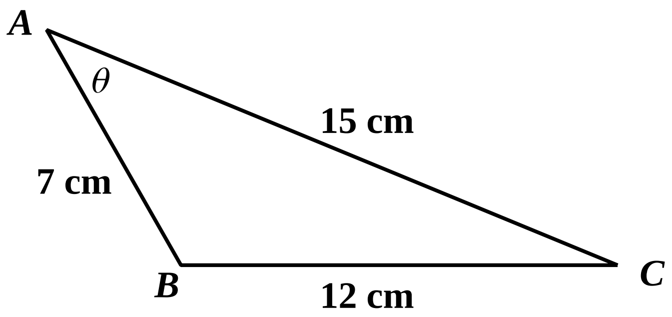
Convert the following angles from degrees into radians.

Write each answer in terms of π

(i) 30° **(ii)** 45° **(iii)** 90°

[3 marks]

Question 6



(i) Determine, in radians, the angle marked θ by using the formula,

$$\cos A \;=\; \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

[3 marks]

(ii) Find the area of $\triangle ABC$ by using the formula,

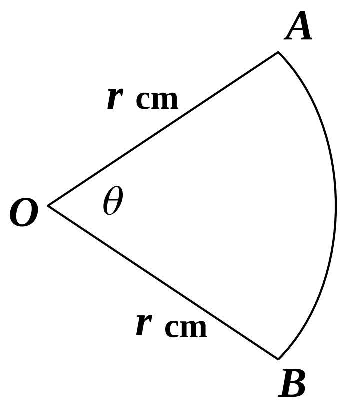
$$Area\ \triangle \;=\; \frac{1}{2}\, b\, c \sin A^c$$

[2 marks]

Question 7

A-Level Examination Question from June 2018, Q3

The diagram shows a sector AOB of a circle with centre O and radius r cm
The angle AOB is θ radians
The area of the sector AOB is 11 cm^2



Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r

[4 marks]