Lesson 2

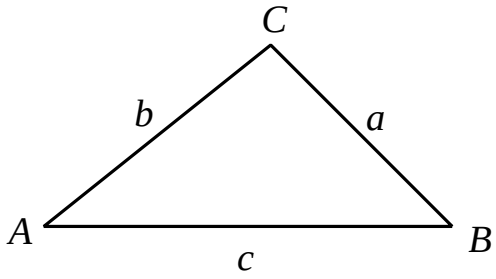
A-Level Pure Mathematics : Year 2

Applications of Trigonometry

2.1 Arc Length and Sector Area Problems

All of our Non-Right Angled Triangle Formulae work equally well whether the units of angle measurement are Radians or Degrees.

Non-Right Angled Triangle Trigonometric Formulae



The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Known : ***One side length & two (and $\therefore$ three) angles***

Seeking : ***Any side length***

The Upside Down Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Known : ***Two side lengths & an excluded angle***

Seeking : ***The other excluded angle***

The Cosine Rule

$$a^2 = b^2 + c^2 - 2\,b\,c\cos A$$

Known : ***Two side lengths & the included angle***

Seeking : ***The third side length***

The Reversed Cosine Rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

Known : ***Three side lengths***

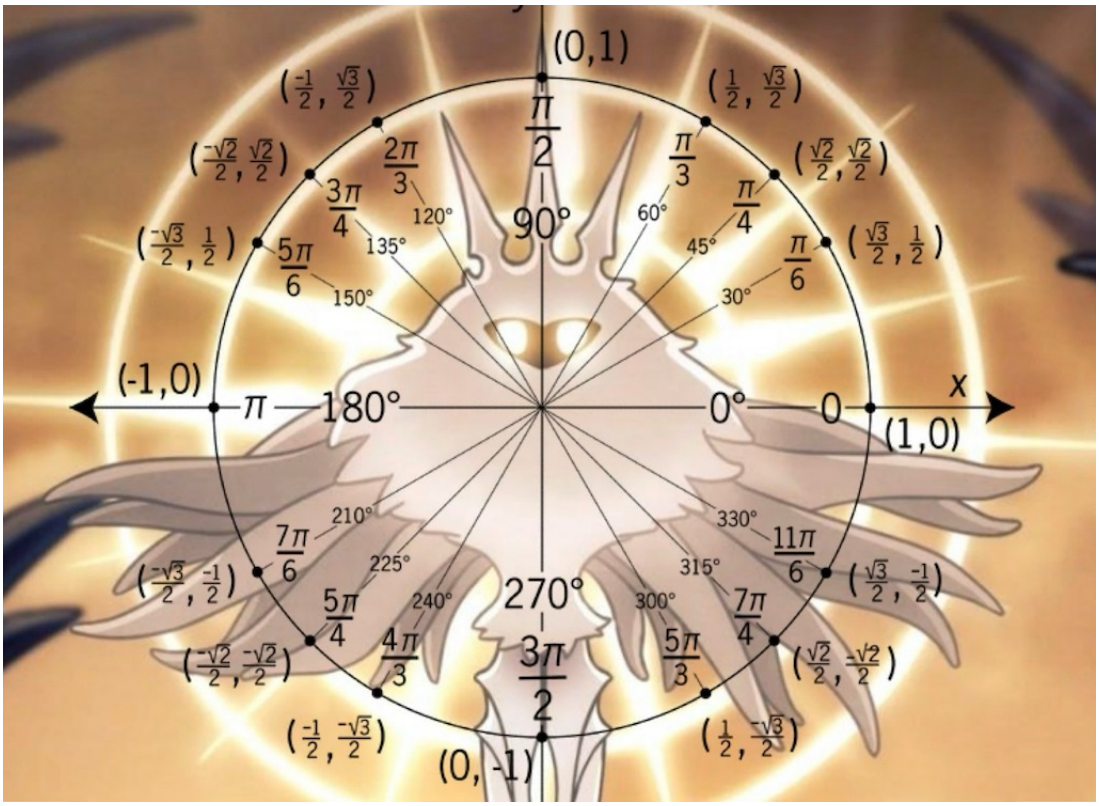
Seeking : ***Any angle***

Useful Area of a Triangle Formula

$$Area\ \Delta = \frac{1}{2}\,a\,b\sin C$$

2.2 Radian, Degrees Conversion Diagram

The following diagram may be of use in getting a feel for the relationship between radians and degrees. Mathematically the key fact to memorise is that $\pi^c = 180°$



2.3 Video Support

Most of the questions in Exercise 2.4 come with a video solution from “Exam Solutions”. It's time consuming to watch lots of video's and you are not expected to do so. After trying a question, should you need help, then watch the video. From experience, students find Question 5(b) tricky.

2.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 45

Here is a reminder of the formulae for Arc Length and Sector Area.
Angles MUST be in radians.

Arc Length and Sector Area

$$\textit{Arc length} \;=\; r\,\theta^c$$

$$\textit{Sector area} \;=\; \frac{1}{2}\,r^2\,\theta^c$$

Question 1

A-Level Examination Question from the C2 Paper, January 2011, Q2

In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm

(a) Find the size of angle C, giving your answer in radians to 3 significant figures

[3 marks]

(b) Find the area of triangle ABC

Give your answer in cm^2 to 3 significant figures

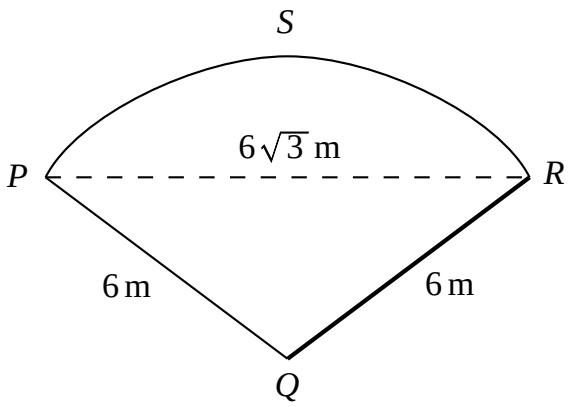
[3 marks]

Need help with Question 1 ?

http://www.NumberWonder.co.uk/v9057/2a.mp4

Question 2

A-Level Examination Question from the C2 Paper, January 2007, Q9



Shown is a plan of a patio.
The patio *PQRS* is in the shape of a sector of a circle with centre *Q* and radius 6 m.

Given that the length of the straight line *PR* is $6\sqrt{3}$ m,

(a) find the exact size of angle *PQR* in radians

[3 marks]

(b) Show that the area of the patio *PQRS* is 12π m^2

[2 marks]

(c) Find the exact area of the triangle *PQR*

[2 marks]

(d) Find, in m^2 to 1 decimal place, the area of the segment *PRS*

[2 marks]

(e) Find, in m to 1 decimal place, the perimeter of the patio *PQRS*

[2 marks]

Need help with Question 2 ?

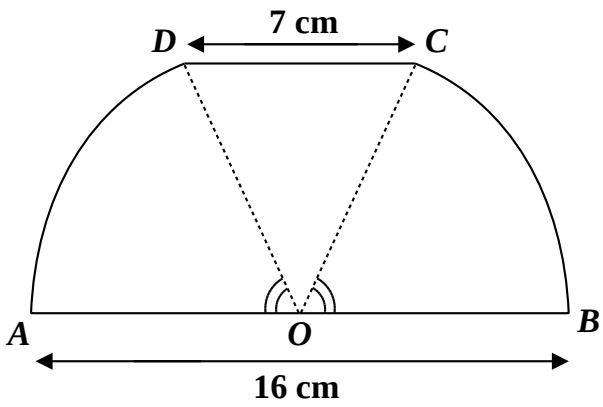
http://www.NumberWonder.co.uk/v9057/2b.mp4 (Part (a))
http://www.NumberWonder.co.uk/v9057/2c.mp4 (Part (b))
http://www.NumberWonder.co.uk/v9057/2d.mp4 (Part (c))
http://www.NumberWonder.co.uk/v9057/2e.mp4 (Part (d))
http://www.NumberWonder.co.uk/v9057/2f.mp4 (Part (e))

Part (a) Part (b) Part (c)

Part (d) Part (e)

Question 3

A-Level Examination Question from the C2 Paper, May 2015, Q4



Shown is a sketch design for a scraper blade.
The blade *AOBCDA* consists of an isosceles triangle *COD* joined along its equal sides to sectors *OBC* and *ODA* of a circle with centre *O* and radius 8 cm.
Angles *AOD* and *BOC* are equal.
AOB is a straight line and is parallel to the line *DC*.
DC has length 7 cm.

(a) Show that the angle *COD* is 0.906 radians, correct to 3 significant figures.

[2 marks]

(b) Find the perimeter of *AOBCDA*, giving your answer to 3 significant figures.

[3 marks]

(c) Find the area of *AOBCDA*, giving your answer to 3 significant figures.

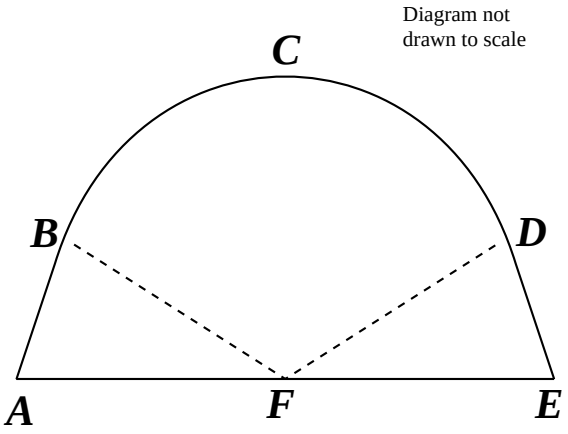
[3 marks]

Need help with Question 3 ?
http://www.NumberWonder.co.uk/v9057/2g.mp4 (Part (a))
http://www.NumberWonder.co.uk/v9057/2h.mp4 (Part (b))
http://www.NumberWonder.co.uk/v9057/2i.mp4 (Part (c))

Part (a) Part (b) Part (c)

Question 4

A-Level Examination Question from the C2 Paper, June 2017, Q4



The sketch represents the cross-section of a large tent, *ABCDEF*
AB and *DE* are line segments of equal length
Angle *FAB* and angle *DEF* are equal
F is the midpoint of the straight line *AE* and *FC* is perpendicular to *AE*
BCD is an arc of a circle of radius 3.5 m with centre at *F*
It is given that

$$AF = FE = 3.7 \text{ m}$$
$$BF = FD = 3.5 \text{ m}$$
$$\text{angle } BFD = 1.77 \text{ radians}$$

Find

(a) the length of the arc *BCD* in metres to 2 decimal places

[2 marks]

(b) the area of the sector *FBCD* in m^2 to 2 decimal places

[2 marks]

(c) the total area of the cross-section on the tent in m^2 to 2 decimal places

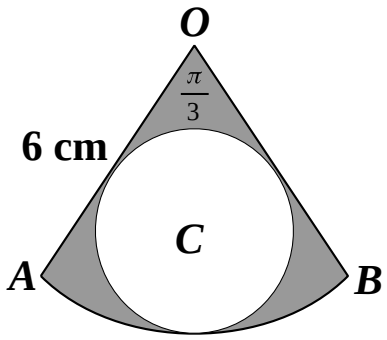
[4 marks]

Need help with Question 4 ?
http://www.NumberWonder.co.uk/v9057/2j.mp4 (All Parts)

All Parts

Question 5

A-Level Examination Question from the C2 Paper, May 2011, Q5



The shape shown is a pattern for a pendant

It consists of a sector OAB of a circle centre O, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$

The circle C, inside the sector, touches the two straight edges, OA and OB and the arc AB as shown.

Find

(a) the area of the sector OAB

[2 marks]

(b) the radius of the circle C

[3 marks]

The region outside the circle *C* and inside the sector *OAB* is shown shaded.

(c) Find the area of the shaded region

[2 marks]

Need help with Question 5 ?

http://www.NumberWonder.co.uk/v9057/2k.mp4 (Part (a))

http://www.NumberWonder.co.uk/v9057/2l.mp4 (Part (b))

http://www.NumberWonder.co.uk/v9057/2m.mp4 (Part (c))

Part (a) Part (b) Part (c)

Question 6

(i) Sketch a circle of radius r, with a chord PQ added.
Within the sector POQ, of angle θ, shade the sector between chord PQ and arc PQ and label its area, A

[2 marks]

(ii) By considering the formula for the area of a sector and the area of a triangle show that

$$A = \frac{1}{2} r^2 (\theta - \sin \theta)$$

[3 marks]