Lesson 4

A-Level Pure Mathematics : Year 2
Applications of Trigonometry

4.1 Trig Equations In Radians

For GCSE many formulae were learnt, such as The Sine Rule and The Cosine Rule. As was seen in Lesson 2, these formulae work just as well whether radians are in use or degrees. Similarly, all of the techniques learnt in the Year 1 A-Level course to solve trigonometric equations work equally well in radians or degrees.

In the A-Level examination, a questions may be presented with the angles in radians along with an expectation that the answer will be given in radians. Such a question can be tackled entirely in radians or, at the start, the angles converted to degrees, the problem then tackled, and the answers in degrees convert to radians. It's crucial not to underestimate how many answers there are in the given interval; the key step where care is needed is the point at which you take *arcsin*, *arccos* or *arctan.*

4.2 Example

Solve the following equation over the interval $0 \leqslant x \leqslant 2\pi$
Give exact answers in terms of π

$$2\cos\left(3x + \frac{\pi}{2}\right) = 1$$

Teaching Video : http://www.NumberWonder.co.uk/v9057/4.mp4

Watch the video.
Write out a solution.

☞

[5 marks]

4.3 Exact values table

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not Defined
	0^c	$\frac{\pi}{6}^c$	$\frac{\pi}{4}^c$	$\frac{\pi}{3}^c$	$\frac{\pi}{2}^c$

4.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 75

Question 1

Solve the following equation over the interval $0 \leqslant x \leqslant 2\pi$

Give exact answers in terms of π

$$2\cos\left(x + \frac{2\pi}{3}\right) = \sqrt{2}$$

[5 marks]

Question 2

Solve the following equation over the interval $0 \leqslant x \leqslant 2\pi$

Give exact answers in terms of π

$$\sqrt{3}\ tan\left(2x + \frac{\pi}{4}\right) = 3$$

[6 marks]

Question 3

A-Level C2 Examination question from May 2011, Q7.

(a) Solve for $0 \leqslant x < 360°$, giving your answers in degrees to 1 decimal place,

$$3 \sin (x + 45°) = 2$$

[4 marks]

(b) Find, for $0 \leqslant x < 2\pi$, all the solutions of

$$2 \sin^2 x + 2 = 7 \cos x$$

giving your answers in radians.

You must show clearly how you obtained your answers.

[6 marks]

Question 4

A-Level C3 (R) Examination question from June 2013, Q6

(i) Use an appropriate double angle formula to show that

$$\text{cosec } 2x \equiv \lambda \text{ cosec } x \text{ sec } x$$

and state the value of the constant λ

[3 marks]

(ii) Solve, for $0 \leqslant \theta < 2\pi$, the equation

$$3 \sec^2 \theta + 3 \sec \theta = 2 \tan^2 \theta$$

You must show all your working. Give your answers in terms of π

[6 marks]

Question 5

A-Level C3 Examination question from June 2012, Q5

(i) Express

$$4\,cosec^2 2\theta - cosec^2 \theta$$

in terms of $sin\ \theta$ and $cos\ \theta$

[2 marks]

(ii) Hence show that

$$4\,cosec^2 2\theta - cosec^2 \theta = sec^2 \theta$$

[4 marks]

(iii) Hence or otherwise solve, for $0 \leqslant \theta < \pi$

$$4\,cosec^2 2\theta - cosec^2 \theta = 4$$

giving your answers in terms of π

[3 marks]

Question 6

A-Level C3 (R) Examination question from June 2014, Q3

(i) **(a)** Show that

$$2\tan x - \cot x = 5\operatorname{cosec} x$$

may be written in the form

$$a\cos^2 x + b\cos x + c = 0$$

stating the values of the constants a, b and c

[4 marks]

(b) Hence solve, for $0 \leqslant x < 2\pi$ the equation

$$2\tan x - \cot x = 5\operatorname{cosec} x$$

giving your answers to 3 significant figures

[4 marks]

(ii) Show that

$$\tan\theta + \cot\theta \equiv \lambda \operatorname{cosec} 2\theta \qquad \theta \neq \frac{n\pi}{2},\ n \in \mathbb{Z}$$

stating the value of the constant λ

[4 marks]

Question 7

A-Level C3 Examination question from June 2010, Q7

(i) Express

$$2 \sin \theta - 1.5 \cos \theta$$

in the form

$R \sin (\theta - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 4 decimal places

[3 marks]

(b) **(i)** Find the maximum value of

$$2 \sin \theta - 1.5 \cos \theta$$

(ii) Find the value of θ, for $0 \leqslant \theta < \pi$ at which this maximum occurs

[3 marks]

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right) \qquad 0 \leqslant t < 12$$

where t hours is the number of hours after midday

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs

[3 marks]

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres

[6 marks]

Question 8

A-Level C3 Examination question from June 2015, Q8

(a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A} \qquad A \neq \frac{(2n + 1)\,\pi}{4} \quad n \in \mathbb{Z}$$

[5 marks]

(b) Hence solve, for $0 \leqslant A < 2\pi$

$$\sec 2A + \tan 2A = \frac{1}{2}$$

Give your answers to 3 decimal places

[4 marks]