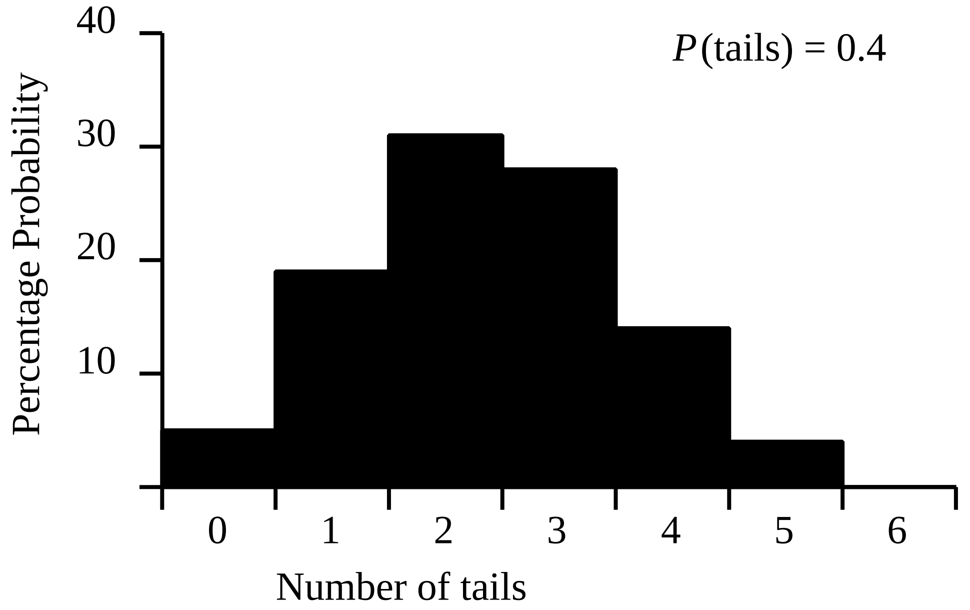
Year 1

~ Statistics ~

STATISTICAL DISTRIBUTIONS

Featuring

The Binomial Distribution



1.1 The Concept of a Random Variable

When a standard six-face dice is rolled, the result can vary at random between six ***discrete*** possible outcomes, 1, 2, 3, 4, 5 or 6

The list of possible outcomes is called the ***sample space***.

We can assign a ***random variable***, X, to the event of rolling the dice.

If the dice is fair,

$$P(X = 1) = \frac{1}{6}$$

$$P(X = 2) = \frac{1}{6}$$

$$P(X = 3) = \frac{1}{6}$$

$$P(X = 4) = \frac{1}{6}$$

$$P(X = 5) = \frac{1}{6}$$

$$P(X = 6) = \frac{1}{6}$$

A far more succinct way of saying this is to write it as a ***probability mass function***

$$P(X = x) = \frac{1}{6},\ x = 1,\ 2,\ 3,\ 4,\ 5,\ 6$$

This is showing how the possible probabilities are distributed over the sample space.
In this case, because all of the probabilities are the same, $\frac{1}{6}$, this particular statistical distribution is known as a ***discrete uniform probability distribution***

For all discrete probability distributions,

$$\sum P(X = x) = 1$$

Example

When two fair coins are tossed, the probability distribution for the number of tails, x, can be described in the table;

x	0	1	2
$P(X = x)$	0.25	0.5	0.25

- Notice that writing out a table like this is an easy alternative way of giving the distribution rather than trying to construct a probability mass function.

Example

A biased four-faced dice with faces numbered 1, 2, 3 and 4 is rolled.
The number on the bottom-most face is modelled as a random variable X
Given that

$$P(X = x) = \frac{x}{k}$$

(i) Find the value of k

(ii) Give the probability distribution of X by completing the following table;

x	1	2	3	4
$P(X = x)$			0.3	

(iii) Find $P(X \geqslant 3)$

(iv) Determine $P(1 < X < 4)$

(v) What is $P(X = 5)$?

1.2 Exercise

Question 1

A discrete random variable X has the probability distribution shown in the table

x	1	2	3	4
$P(X = x)$	0.33	0.17	k	0.24

Find the value of k

Question 2

A discrete random variable X has the probability distribution shown in the table

x	1	2	3
$P(X = x)$	$0.4 - a$	a	$0.3 + 2a$

Find the value of a

Question 3

A biased four-faced dice with faces numbered 1, 2, 3 and 4 is rolled.
The number on the bottom-most face is modelled as a random variable X
Given that

$$P(X = x) = \frac{k}{x}$$

(i) Find the value of k

(ii) Give the probability distribution of X by completing the following table;

x	1	2	3	4
$P(X = x)$			0.16	

(iii) Find $P(X \geqslant 3)$

(iv) Determine $P(2 \leqslant X < 4)$

(v) What is $P(X < 1)$?

Question 4

The discrete random variable X has the following probability distribution

x	1	5	9
$P(X = x)$	a	b	c

It is known that

$$P(X < 4) = P(X > 4)$$

and

$$P(X \leqslant 5) = 2P(X > 5)$$

Find the values of a, b and c

Question 5

When three fair coins are tossed, the probability distribution for the number of tails, x, can be described in a table;

x	0	1	2	3
$P(X = x)$				

Complete the table.

Question 6

The discrete random variable X has a probability function

$$P(X = x) = \begin{cases} k & x = 1,\ 3 \\ k(x - 1) & x = 2,\ 4 \end{cases}$$

where k is a constant

(i) Find the value of k

(ii) Find $P(X > 1)$

Question 7

The discrete random variable X has a probability function

$$P(X = x) = \begin{cases} \left(\frac{1}{2}\right)^x & x = 1,\ 2,\ 3,\ 4,\ 5 \\ k & x = 6 \end{cases}$$

where k is a constant

Find the value of k

Question 8

A fair coin is tossed repeatedly until a head appears.
The random variable T represents the number of tosses.

Complete the following table to show the probability distribution of T

x	1	2	3	4	5	> 5
$P(T = x)$						

Question 9

The discrete random variable X has a probability function

$$P(X = x) = \begin{cases} 0.1 & x = 1,\ 2 \\ k & x = 3,\ 4 \\ 0.2 & x = 5 \end{cases}$$

where k is a constant

(i) Find the value of k

(ii) Construct a table giving the probability distribution of X

(iii) Find $P(2 \leqslant X < 5)$

Question 10

A biased coin is tossed until a head appears or it is tossed four times where

$$P\ (\ Head\ )\ =\ \frac{2}{3}$$

The random variable *T* represents the number of tosses

Show in a table the probability distribution of *T*

Question 11

In the game of Monopoly two dice are spun and the sum of the pips showing found.

Find a probability mass function for the distribution of the form

$$P\ (\ X\ =\ x\ )\ =\ \left\{ \begin{array}{ll} k\ (\ x\ -\ a\ ) & x\ =\ 2,\ 3,\ 4,\ 5,\ 6,\ 7 \\ & \\ k\ (\ b\ -\ x\ ) & x\ =\ 8,\ 9,\ 10,\ 11,\ 12 \end{array} \right\}$$

where *a*, *b* and *k* are constants that you have determined