

Lesson 2

A-level Applied Mathematics Statistical Distributions : Year 1

2.1 Investigating Binomial Probability

Binomial Probability means 'two state' probability, and thus far the obvious real life situation that this models is the flipping of a coin.

2.2 A Coin Flipping Example

A biased coin is weighted such that it has a probability of 0.36 of landing tails. It is flipped 8 times. What is the probability that it lands tails exactly twice ?

H H H H H H T T

To answer this question we begin by wanting to know how many different ways 2 tails and 6 heads can occur.

H H H H H T H T

H H H H H T T H

Method 1 : List them.

Observe that the list has 28 entries.

This takes a long time and is tricky to get correct.

H H H H T H H T

H H H H T H T H

H H H H T T H H

Method 2 : Use Pascal's Triangle.

observe that Row 8, Column 2 is 28.

H H H T H H H T

H H H T H H T H

H H H T H T H H

H H H T T H H H

Less time but still a big job to get just one number.

1									
1	1								
1	2	1							
1	3	3	1						
1	4	6	4	1					
1	5	10	10	5	1				
1	6	15	20	15	6	1			
1	7	21	35	35	21	7	1		
1	8	28	56	70	56	28	8	1	

H H T H H H H T

H H T H H H T H

H H T H H T H H

H H T H T H H H

H H T T H H H H

Method 3 : Use a calculator.

$${}^8C_2 = 28$$

Having got the 28....

Each of the 28 items in the list will have the same value of $0.36^2 \times 0.64^6$

H T H H H H H T

H T H H H H T H

H T H H H T H H

H T H H T H H H

H T H T H H H H

H T T H H H H H

Thus the calculation is;

$$28 \times 0.36^2 \times 0.64^6 = 0.249$$

$$= 28 \times 0.1296 \times 0.0687$$

$$= 0.249 \text{ (give answers to 3 decimal places)}$$

T H H H H H H T

T H H H H H T H

T H H H H T H H

T H H H T H H H

T H H T H H H H

T H T H H H H H

T T H H H H H H

So, when this biased coin is flipped 8 times, there is a 25% probability that it will land tails exactly twice.

2.3 Exercise

Question 1

Here is Pascal's Triangle, left justified.

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
```

- (i) Circle the entry in Row 7, Column 5.
Notice this is in the 8th row and the 6th column !
- (ii) Use your calculator to determine 7C_5
This should be your part (i) answer.

Question 2

Here is Pascal's Triangle, left justified.

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
```

- (i) Circle the entry in Row 4, Column 0
- (ii) Circle the entry in Row 5, Column 5
- (iii) Circle the entry in Row 9, Column 3
- (iv) Find a solution pair (n, r) to

$${}^nC_r = 70$$

Question 3

Use your calculator to determine $^{13}C_5$

Question 4

A coin is flipped 6 times.

- (i) Describe which calculator buttons you would press in order to determine the number of ways exactly 2 tails could be obtained ?

- (ii) Write out enough of Pascal's Triangle so that you can then draw a circle around the number in it, corresponding to your part (i) answer.

Question 5

A coin is flipped 20 times.

In how many ways can exactly 13 tails be obtained ?

HINT : Use a calculator.

Question 6

A coin is flipped 40 times.

In how many ways can exactly 5 heads be obtained ?

Question 7

A biased coin is weighted such that it has a probability of 0.45 of landing tails. It is flipped 8 times. What is the probability that it lands tails exactly thrice ? Give your answer to 3 decimal places.

Question 8

A typist has a probability of 0.99 of typing each letter in a sentence correctly. What is the percentage probability of exactly two mistakes in a sentence containing 180 letters, if mistakes are made at random ? Give your answer to 3 decimal places.

Question 9

In a box of Smarties™ there are eight different colours which normally occur in equal proportions. Sebastian is given 24 Smarties™, and blue ones are his favourite. Assume these come from a very large box.

- (i) How many blue Smarties™ would he expect to get ?
- (ii) What is the probability that he gets this number ?
- (iii) What is the probability that he gets fewer than expected ?
- (iv) What is the probability that he gets more than expected ?
- (v) Explain why the assumption was made that the 24 Smarties™ given to Sebastian came from a very large box ?

Question 10

- (i) Determine 6C_2
- (ii) Determine 6C_4
- (iii) Explain, with the help of Pascal's Triangle, why ${}^6C_2 = {}^6C_4$
- (iv) Explain, in your own words, why ${}^nC_0 = {}^nC_n$
- (v) Explain, in your own words, why ${}^nC_1 = {}^nC_{n-1}$
- (vi) What formula can be written down for nC_m that generalises the observations made above ?

The next question is about obtaining the probability distribution curve for a simple coin flipping situation.

Question 11

A biased coin is weighted such that it has a probability of 0.4 of landing tails. It is flipped 6 times.

- (i) Show that the probability of exactly 4 tails being obtained is 0.138.
 i.e. About 14% probable.
- (ii) Work out the probability of exactly 0, 1, 2, 3, 5 and 6 tails being obtained.
 Present your solutions in the table below.

N° of tails	0	1	2	3	4	5	6
Probability %					14		

- (iii) Present your table of results as a bar chart.

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com