3.1 Graphing The Binomial Distribution

The final question of the previous exercise was concerned with obtaining the probability distribution curve for a simple coin flipping situation.
Here is a recap of the question along with the answer;

A biased coin is weighted such that it has a probability of 0.4 of landing tails.
It is flipped 6 times.

(i) Show that the probability of exactly 4 tails being obtained is 0.138.
i.e. About 14% probable.

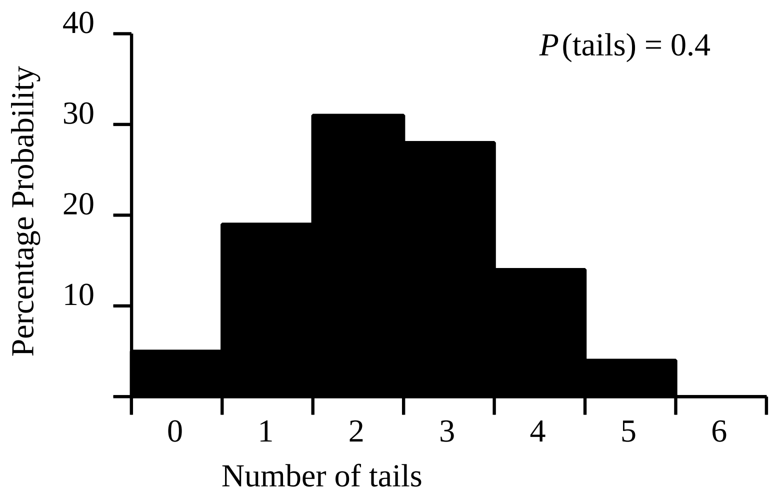
$$\begin{aligned} P(4\,tails) &= {}^6C_4 \times 0.4^4 \times (1 - 0.4)^{6-4} \\ &= 15 \times 0.4^4 \times 0.6^2 \\ &= 0.138 \end{aligned}$$

(ii) Work out the probability of exactly 0, 1, 2, 3, 5 and 6 tails being obtained.
Present your solutions in the table below.

N° of tails	0	1	2	3	4	5	6
Probability	5%	19%	31%	28%	14%	4%	0%

Rounding errors mean we've got 101% but as the table is being used to produce a bar chart, we'll not worry about that !

(iii) Present your table of results as a bar chart.



A key observation is that the bar chart is lop-sided.
It peeks before the half way value of 3 tails and 3 heads.
The mathematical word for this lopsidedness is *skew*

3.2 Binomial Probability Formula

The calculation being done repeatedly in this work can be written as a formula,

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$$P(\,r\ successes\,) = {}^{n}C_{r}\ p^{r}\ q^{n-r}$$

where *n is the number of trials* (how many times the coin was flipped)
r is the number of successes (how many times the coin is to land tails)
p is the probability of success (the probability of landing tails)
q is the probability of failure (the probability of landing 'not tails')

Of course, it comes from the context of the question what is considered success and what is considered failure.

However, in binomial probability;

$$q = 1 - p$$

3.3 Exercise

Question 1
A biased coin is weighted such that it has a probability of 0.3 of landing tails.
It is flipped 8 times.

(i) Show that the probability of exactly 5 tails being obtained is 0.047.
i.e. About 5% probable.

(ii) Work out the probability of exactly 0, 1, 2, 3, 4, 6, 7 and 8 tails being obtained and present your solutions in the table on the next page.

N° of tails	0	1	2	3	4	5	6	7	8
Probability %						4.7			

(iii) Present your table of results as a bar chart.

Question 2

Additional Mathematics, June 2010, Q4.

In a game 4 fair dice are thrown.

Calculate the probability that

(i) no six is thrown

[2 marks]

(ii) at least 2 sixes are thrown.

(*Be careful; at least 2 means* ***2 or more***)

[4 marks]

Question 3

Additional Mathematics, Specimen Paper, Q13.

China cups are packed in boxes of 10.

It is known that 1 in 8 are cracked.

Find the probability that in a box of 10, chosen at random,

(a) no cups are cracked,

[3 marks]

(b) exactly 1 cup is cracked,

[4 marks]

(c) exactly 2 cups are cracked,

[2 marks]

(d) at least 3 cups are cracked

[3 marks]

Question 4
Additional Mathematics, June 2004, Q9
The probability that I am late for work on any given day is 0.1
Being late on one day is independent of any other day.
Find the probability that in a week of 5 working days I am late at least twice.
Give your answer correct to 4 decimal places.

[5 marks]

Question 5

Additional Mathematics, June 2005, Q5

In a large batch of glasses, 14 % are defective.
From this batch 8 glasses are selected at random.
Calculate which is more likely;

(A) None of the glasses is defective

(B) At least two of the glasses are defective.

[7 marks]

Question 6

Additional Mathematics, June 2008, Q4.

Glass marbles are produced in two colours, red and green, in the proportion 7:3 respectively. From a large stock of the marbles, 5 are taken at random.

Find the probability that

(i) all 5 are red

[2 marks]

(ii) exactly 3 are red

[3 marks]

Question 7

Additional Mathematics, June 2011, Q11.

Eggs are delivered to a supermarket in boxes of 6.

For each egg, the probability that it is cracked is 0.05 independently of other eggs.

Find the probability that

(i) in one box there are no cracked eggs

[2 marks]

(ii) in one box there is exactly 1 cracked egg.

[4 marks]

The manager checks the eggs as follows;

- ◇ He takes a box at random from the delivery.
- ◇ He accepts the whole delivery if this box contains no cracked eggs.
- ◇ He rejects the whole delivery if the box contains 2 or more cracked eggs.
- ◇ If the box contains 1 cracked egg then he chooses another box at random.
- ◇ He accepts the delivery only if this second box contains no cracked eggs.

(iii) Find the probability that the delivery is rejected.

[6 marks]