4.1 Binomial Distribution Calculation

When conducting an experiment or survey a random variable X can be defined to represent the number of successful trials.

X can be modelled by a binomial distribution, $B(n, p)$ if:

- ◇ n (the number of trials) are fixed and independent
- ◇ there are precisely two possible outcomes, success and failure
- ◇ p the probability of success, is fixed

The probability of failure, q, is then given by $q = 1 - p$

and X has probability mass function;

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

4.2 Example

5 % of Smarties™ are blue.
What is the probability that in a tube of 45 Smarties™, exactly 2 are blue ?

Calculator Buttons for CASIO fx-991EX ClassWiz
MENU : 7 : 4 : 2 : 2 : = : 45 : = : 0.05 : = : =

4.3 Exercise

Question 1

An ordinary six sided dice is rolled 13 times.
What is the probability of rolling a 6 thrice ? (thrice = three times)

Question 2

In a class of 16 children;

(i) What is the probability that exactly 3 children were born on a Monday ?

(ii) What is the probability that exactly 3 children in a were born on a weekend ?

(iii) What is the probability that exactly 13 children were born on a weekday ?

Question 3

A general knowledge quiz has 16 questions.
Each question has four options of which one only is the correct answer.
An irritating boy ***goes monkey***; he attempts the quiz by pure guesswork.

(i) What is the percentage probability that he gets all of the questions wrong ?

(ii) How many questions would you expect him to get right ?
This is the ***monkey number*** for the quiz.

(iii) What is the percentage probability that he gets the ***monkey number*** of questions correct ?

(iv) Is this distribution's skew positive, negative or symmetrical ?

Question 4
Given the distribution $X \sim B(11, 0.3)$, find $P(X = 3)$

Question 5
Given the distribution $X \sim B(4, 0.6)$

Find **(i)** $P(X = 0)$

(ii) $P(X = 1)$

(iii) $P(X < 2)$

(iv) $P(X > 1)$

Question 6
In a game of darts, the probability that I hit the bullseye is 0.2.
(I'm not skilled; I hit the bullseye at random !)
I have six darts.

What is the probability that I,
(i) don't hit the bullseye ?

(ii) hit the bullseye once ?

(iii) hit the bullseye twice ?

(iv) hit the bullseye with more than once ?

Question 7

A typist has a probability of 0.98 of typing a letter correctly.
He makes his mastikes at rondam.
He types a sentence containing 200 letters.

What is the probabilitee that he makes more than one mixtakes ?

Is he a good typist ?

Question 8

Given the distribution $X \sim B\ (\ 5,\ 0.4\ )$

Complete the following table accurate to four decimal places,

x	$P\ (\ X = x\ )$	Cumulative $P\ (\ X \leqslant x\ )$
0		
1		
2		
3		
4		
5		

From the table find $P(\ X \leqslant 3\ )$

Question 9

Here is a cumulative probability table for $X \sim B(10, 0.3)$.

x	Cumulative $P(X \leqslant x)$
0	0.0282
1	0.1493
2	0.3828
3	0.6496
4	0.8497
5	0.9527
6	0.9894
7	0.9984
8	0.9999
9	1.0000

From the table find;

(i) $P(X \leqslant 4)$ **(iv)** $P(X \geqslant 4)$

(ii) $P(X < 4)$ **(v)** $P(X > 4)$

(iii) $P(X = 4)$ **(vi)** $P(X = 6)$

Check answers **(iii)** and **(vi)** by direct calculation.