

Lesson 5

A-level Applied Mathematics Statistical Distributions : Year 1

5.1 Using The Printed Binomial Cumulative Distribution Tables

Suppose we have the distribution $X \sim B(5, 0.25)$

and a set of BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION tables...

- (i) Copy the appropriate part of the supplied tables into the following table.

x	$P(X = x)$	Cumulative $P(X \leq x)$
0		
1		
2		
3		
4		
5		

- (ii) Complete the above table.

- (iii) Use the formula given below to check the result for $P(X = 3)$

$$P(X = x) = \binom{n}{r} p^r q^{n-r}$$

- (iv) Using only the numbers from the Cumulative column, find

(a) $P(X \leq 3)$

(b) $P(X < 3)$

(c) $P(X > 2)$

(d) $P(X \geq 4)$

(e) $P(X = 2)$

5.2 A Useful Result

The numbers in the supplied Binomial Cumulative Distribution Tables are all of the form $P (X \leq x)$

Take care when a question asks for $P (X \geq x)$

It may help to keep in mind that,

$$P (X \geq k) = 1 - P (X \leq k - 1)$$

5.3 Exercise

Question 1

The random variable $X \sim B(20, 0.35)$

Use the printed binomial cumulative distribution tables to find;

(i) $P (X < 8)$

(ii) $P (X > 6)$

(iii) $P (X \geq 4)$

(iv) $P (X \leq 9)$

(v) $P (2 \leq X \leq 7)$

Question 2

Ten fair coins are tossed and the total number of heads showing is recorded.

Use the printed binomial cumulative distribution tables to find the probability of;

(i) at least 2 heads

(ii) more heads than tails

Question 3

For a particular type of plant 35 % have red flowers.

A garden centre sells these plants in trays of 15 plants of mixed colours.

A tray is selected at random.

Use the printed binomial cumulative distribution tables to find the probability that the number of plants with red flowers in this tray is;

(i) less than 4

(ii) at most 2

(iii) between 3 and 5 (inclusive)

Question 4

A dice is biased so that the probability of it landing on a six is 0.3

Bilal rolls the dice 15 times.

(i) State any assumptions that are required to model the number of sixes as a binomial distribution.

(ii) Find the probability that Bilal rolls exactly 4 sixes.

(iii) Find the probability that Bilal rolls two or fewer sixes.

Question 5

The random variable $X \sim B(30, 0.25)$

Use the printed binomial cumulative distribution tables to find;

(i) $P (X \geqslant 9)$

(ii) $P (6 < X \leqslant 11)$

Question 6

A factory produces a LCD screens for electronic calculators.

From experience it is known that 5 % of the LCD screens are defective.

A quality control officer regularly inspects a random sample of 40 LCD screens.

(i) How many defective LCD screens are expected in a sample of 40 ?

Find the probability that the next sample contains;

(ii) fewer than 2 defective LCD screens

(iii) more than 5 defective LCD screens

Question 7

A random variable X is binomially distributed. In fact, $X \sim B(12, 0.45)$

Find

(i) $P (X \leq 0)$

(ii) $P (X \leq 1)$

(iii) $P (X \leq 2)$

(iv) $P (X \leq 3)$

What is the largest value of k such that $P (X \leq k) < 0.05$

Question 8

A coin is biased with probability 0.25 of landing heads when tossed.

The coin is tossed 50 times.

Use the printed binomial cumulative distribution tables to find the probability that no more than 8 heads are obtained.

Question 9

Given that $X \sim B(25, 0.40)$ find the largest value of k such that $P(X \leq k) < 0.05$

Question 10

A random variable X is binomially distributed. In fact, $X \sim B(50, 0.40)$

Find

(i) $P(X \leq 25)$

(ii) $P(X \leq 26)$

(iii) $P(X \leq 27)$

(iv) $P(X \leq 28)$

What is the largest value of k such that $P(X \leq k) < 0.95$?

What is the smallest value of m such that $P(X \geq m) \leq 0.05$?