Lesson 6

6.1 Binomial Cumulative Distribution Tables From A Calculator

Suppose we have the distribution $X \sim B\,(37, 0.27)$

i.e. n the number of trial is 37
p the probability of success is 0.27

If you reach for a set of printed Binomial Cumulative Distribution Tables, disappointment will follow because the tables jump from $n = 30$ to $n = 40$ missing out what you seek, $n = 37$
Similarly, the tables jump from $p = 0.25$ to $p = 0.30$ missing out $p = 0.27$

So, examination questions can force you to have to abandon the printed tables and test your ability to extract the required information from your statistics calculator instead.

6.2 Example

For the distribution $X \sim B\,(37, 0.27)$ find

(i) $P\,(X \leqslant 9)$

(ii) $P\,(X \geqslant 12)$

(iii) $P\,(4 < X \leqslant 8)$

Note : The examination may also ask questions that are far easier to answer using the printed tables than a calculator
$\therefore$ You need to be competent at using both methods !

Buttons for **(i)** : MENU : 7 : ∇ : 1 : 2 : 9 : = : 37 : = : 0.27 : = : = : to get 0.439

6.3 Exercise

Question 1

For the random variable

$$X \sim B\left(15, \frac{1}{3}\right)$$

(i) Find $P(X \leqslant 5)$

(ii) Determine $P(X \geqslant 7)$

(iii) Calculate $P(3 \leqslant X < 9)$

Question 2

A fair dice is to be rolled 48 times.

(i) On average, how many rolls of "5" are expected

(ii) Find the probability that fewer than 6 rolls of "5" occur

(iii) Find the probability that at most 12 rolls of "5" occur

Question 3

Records kept in a hospital show that 7 out of every 10 patients who visit the accident and emergency department have to wait more than four hours.
Find, to 3 decimal places, the probability that of the first 12 patients who come to the accident and emergency department more than 9 will have to wait more than four hours.

Question 4

In this question it's easiest to use your statistics calculator for parts (i) and (ii) and then the printed tables for part (iii) where you are looking for the lowest cumulative probability that's above 0.95 That is 1 – 0.05

A spinner is designed so that the probability it lands on red is 0.3.
Felix has 12 spins.
Find the probability that Felix obtains;

(i) no more than 2 reds

(ii) at least 5 reds

Felix decides to use this spinner for a class competition.
He wants the probability of winning a prize to be less than 5 %
Each member of the class will have 12 spins and the number of reds will be recorded.

(iii) Find how many reds are needed to win a prize

Question 5

A factory produces a component for the motor trade and 5% of the components are defective.

Grace, a formidable quality control officer inspects a random sample of 50 components

Find the probability that the next sample contains;

(i) fewer than 2 defectives

[1 mark]

(ii) more than 5 defectives

[2 marks]

Grace will stop production if the number of defectives in the sample is **greater than** a certain value *d.* Given that Grace stops production less than 5% of the time

(iii) find the smallest value of *d*

[2 marks]

Question 6

Ed rolls a fair dice repeatedly until he rolls a "6".

(This is to delay his departure from the statistics lesson so that he avoids an accident he was destined to have had he left on time; I worry about Ed)

Calculate the smallest number of rolls such that the probability of his getting at least one "6" exceeds 0.99

HINT : If you are stuck, make progress with some Trial and Improvement

Guess that $n = 10 \Rightarrow X \sim B\left(10, \frac{1}{6}\right) \Rightarrow$ Work out $P\left(X \geqslant 1\right)$

Question 7

(i) State clearly the conditions under which it is appropriate to assume that a random variable has a binomial distribution.

[2 marks]

A door-to-door canvasser tries to persuade people to have a certain type of double glazing installed. The probability that his canvassing at a house is successful is 0.05

(ii) Find the probability that he will have at least 2 successes out of the first 10 houses he canvasses.

[2 marks]

(iii) Calculate the smallest number of houses he must canvass so that the probability of his getting at least one success exceeds 0.99

HINT : If you are stuck, make progress with some Trial and Improvement Guess that $n = 80 \Rightarrow X \sim B(80, 0.05) \Rightarrow$ Work out $P(X \geqslant 1)$

[4 marks]