Applied A-Level Mathematics
Mechanics
Year 2

DYNAMICS II

PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

AUCTORE

ISAACO NEWTONO,

EQUITE AURATO.

EDITIO ULTIMA

AUCTIOR ET EMENDATIOR.

AMSTÆLODAMI

SUMPTIBUS SOCIETATIS,

MDCCXIV.

D Y N A M I C S II

Lesson 1

A-Level Applied Mathematics
Mechanics : Dynamics II : Year 2

1.1 Newton's Laws of Motion

Kinematics is the study of the motion of a particle without regard to any forces.
However, we know from experience that

- ◇ to make a stationary object move,
- ◇ to speed up one that is already moving,

or ◇ to slow one down,

a force needs to be applied.

Dynamics is the study of motion when any forces acting are taken into account.

Newton worked out a great way to think about force and motion which is summarised by his famous ***three laws of motion***.

Newton's Laws of Motion	
1st Law	An object will remain at rest or will continue to move in a straight line at a constant velocity unless it is acted upon by a resultant force.
2nd Law	$F = m\,a$
3rd Law	Every action has an equal and opposite reaction.

1.2 Friction

If a crate is dragged across a frozen lake, less force is needed than if it were dragged along a bumpy concrete path.
The ***friction*** (i.e. the resistance to motion) is different between the two situations, and clearly depends upon the ***nature of the surface***.
In exam questions, if a surface is described as being ***smooth***, then the friction is modelled as being negligible and can be ignored.
However, a surface described as being ***rough*** is one in which the friction is significant and so is to be taken into account.

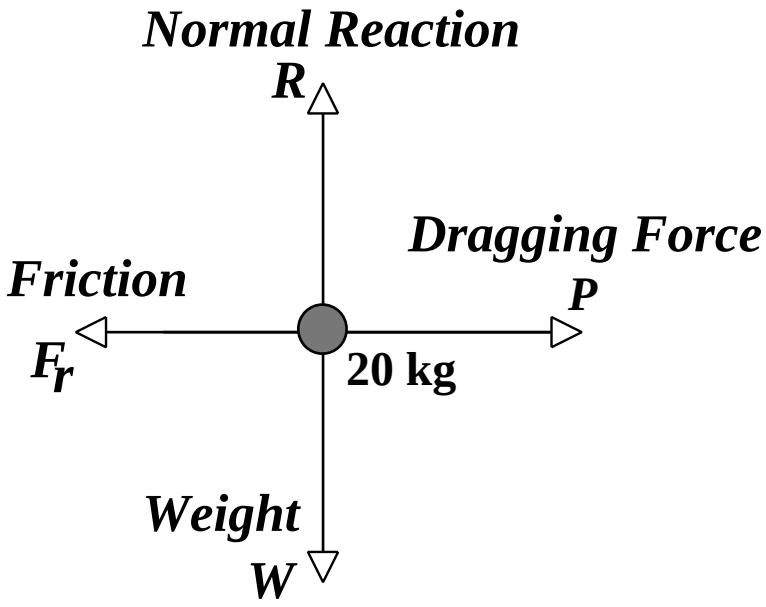
When friction is under consideration, think of it as the 'spoilsport' force.
It tries to stop things happening !

Friction always acts to oppose motion

1.3 Getting A Grip on Friction

Example

A force P pulls a crate of mass 20 kg along a rough level road.
The crate is accelerating at 0.4 m s^{-2}
The coefficient of friction, μ, is 0.7



Find **(i)** The normal reaction, R.
(ii) The friction force, F.
(iii) The force dragging the crate forward, P.

Understanding The Question

There's quite a lot to explain about this question, before trying to answer it.

Firstly, note that F_r is being used to denote friction.
That's to make if clear that it's a different F to the F in the formula $F = m\,a$.

Secondly, note the symbol μ, the coefficient of friction.
This is a measure of how rough the surface is; typically between 0.2 and 0.8.
For ice, μ might be around 0.1.
For toffee covered sandpaper, μ might be around 23.
(Notice that μ can be more than 1)
The simplest way to mathematically model friction is to say it varies directly with the weight of the crate. This seems reasonable; a heavier crate is harder to drag along than a lighter crate.
Thus, *friction is directly proportional to weight.*

$$F_r \propto W$$

Very soon the pulling and pushing of crates up and down hills will be considered, where only part of the weight contributes to the friction effect.
So it's more useful to say that

$$Friction \quad \propto \quad Normal\ Reaction$$

or

$$F_r \propto R$$

It is the constant of this proportionality that we call the coefficient of friction.
Thus

$$F_r = \mu R$$

Now to answer the question:

1.4 Exercise

Where needed, take the value of the acceleration due to gravity, g, to be 9.8 m s^{-2}

Question 1

A crate of mass 35 kg is being pulled along a rough level surface for which the coefficient of friction is 0.6

The crate is accelerating at 0.25 m s^{-2}

Find **(i)** The normal reaction, R

(ii) The friction force, F_r

(iii) The force dragging the crate forward, P

Question 2

A force of 500 N is dragging a 68 kg crate over a rough level surface for which the coefficient of friction is 0.64

Find **(i)** The normal reaction, R

(ii) The friction force, F_r

(iii) The acceleration, a

Question 3

A force of 20 N is dragging a 12 kg box over a rough, level surface.
The box is accelerating at 0.6 m s^{-2}

Find **(i)** The normal reaction, R

(ii) The friction force, F_r

(iii) The coefficient of friction, μ

Question 4

A force of 400 N is dragging a crate over a rough, level surface for which the coefficient of friction is 0.4
The crate is accelerating at 0.3 m s^{-2}

What is the mass, in kg, of the crate ?

Question 5

A particle of mass 4 kg is projected across a horizontal plane with a speed of 7 m s^{-1} If it comes to rest in a distance of 10 m, find the magnitude of the frictional force acting on the particle and the coefficient of friction between the particle and the plane.

Question 6

A body of mass 5 kg is at rest on a horizontal plane for which the coefficient of friction is 0.3. A horizontal force of 21 N is applied to the body. If after 3.5 seconds this force ceases to act, find the further time that elapses before the particle comes to rest.

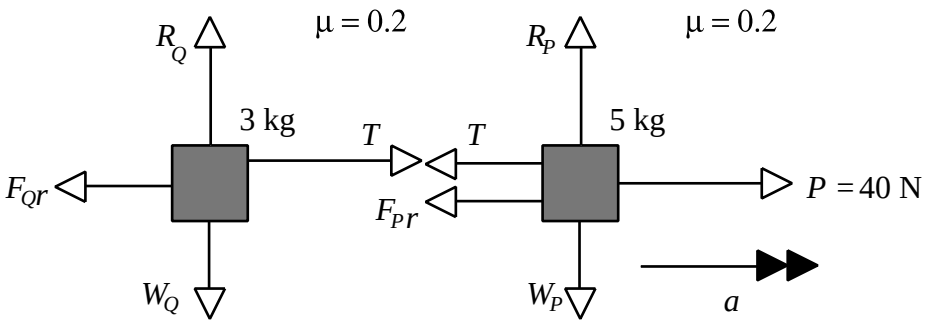
Question 7

Two particles *P* and *Q*, of masses 5 kg and 3 kg respectively, are connected by a light inextensible string. Particle *P* is pulled by a horizontal force of magnitude 40 N along a rough horizontal plane. The coefficient of friction between each particle and the plane is 0.2. The string is taut.

(i) Find the acceleration of each particle.

(ii) Find the tension in the string.

(iii) Explain how the modelling assumptions that the string is light and inextensible have been used in your solution.



Question 8

A vehicle of mass 2 tonnes is travelling along a straight horizontal road at 90 km h^{-1}
It is brought to rest by a constant force of F newtons in a distance of 0.5 km
Find

(i) the time taken for the car to come to rest,

(ii) the force, F