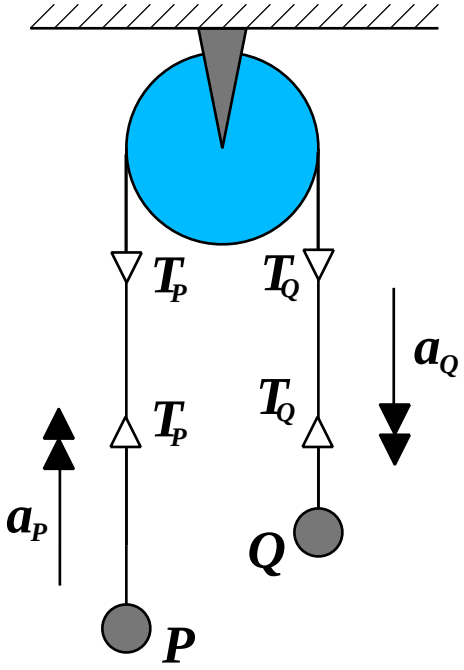
Lesson 5

A-Level Applied Mathematics
Mechanics : Dynamics II : Year 2

5.1 Pulley Power

In the Year 1 course, simple problems involving a pulley were encountered. Typically, the system analysed involved two masses connected via a light and inextensible string, that passed over a pulley.



The Perfect Pulley System : Three Key Facts

1. The tensions in the rope on either side of the pulley are equal.

$$T_P = T_Q$$

This arises from assuming that the pulley is frictionless.

2. The magnitude of the accelerations of the two masses are equal.

$$a_P = a_Q$$

This arises from assuming the string is inextensible.

3. The acceleration of each mass is constant as time passes.
To see why, imagine that the string were a heavy chain. Then, as its weighty links passed over the pulley, mass would be transferred from one side of the system to the other. This would cause the acceleration of each particle to vary.
(Also the tension would to different at different places in the string)

5.2 Example

Two particles *P* and *Q* of masses 5 kg and 10 kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. *P* rests on the inclined plane and *Q* hangs on the edge of the plane with the string vertical and taut.

The plane is inclined to the horizontal at an angle θ where $\tan\theta = \frac{3}{4}$

The coefficient of friction between *P* and the plane is 0.2.

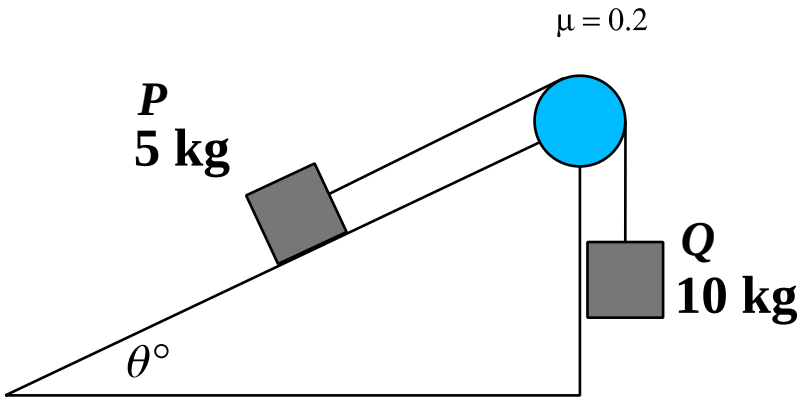
The system is released from rest.

(a) Find the normal reaction for *P*.

(b) By solving simultaneous equations, or otherwise, find

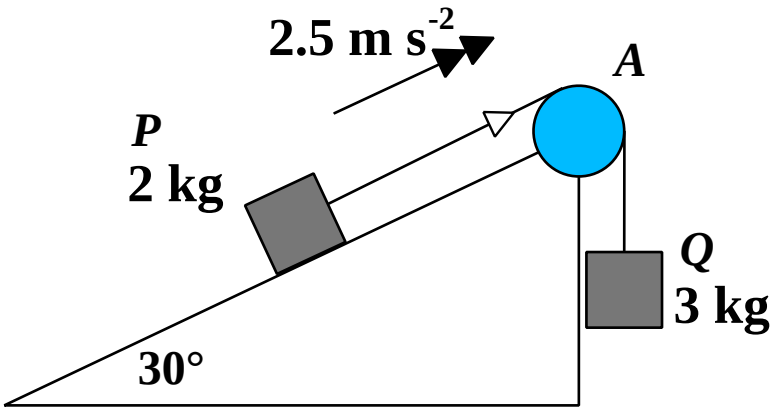
(i) the acceleration of the system

(ii) the tension in the string.



5.3 Exercise

Question 1



Two particles *P* and *Q* of mass 2 kg and 3 kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane plane inclined to the horizontal at an angle of 30°. Particle *P* is held at rest on the inclined plane and *Q* hangs freely with the string vertical and taut. Particle *P* is released and it accelerates up the plane at 2.5 m s^{-2}

(a) Find the tension in the string

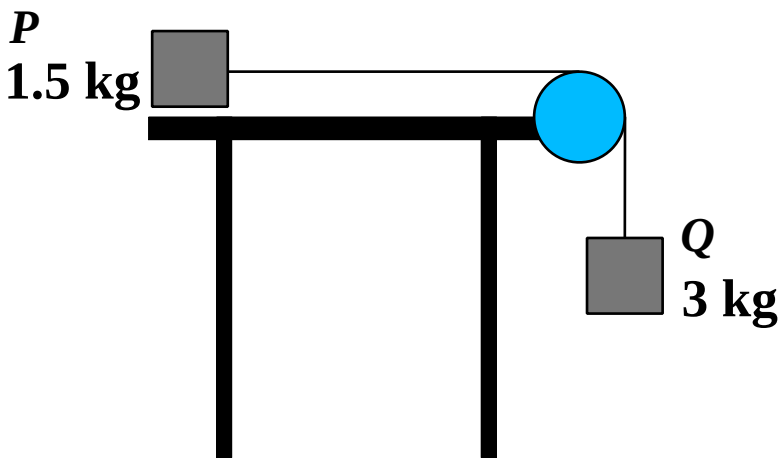
[2 marks]

(b) Determine the coefficient of friction between *P* and the plane

[4 marks]

Question 2

A-Level Examination Question from June 2016, M1, Q8



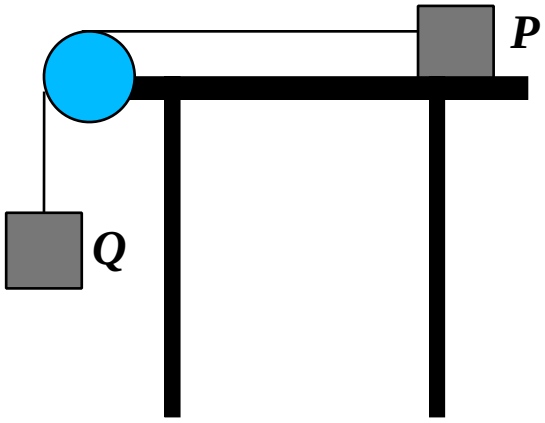
Two particles P and Q, of masses 1.5 kg and 3 kg respectively.
The particles are attached to the ends of a light inextensible string.

The coefficient of friction between P and the table is $\frac{1}{5}$

The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle Q hangs freely at rest vertically below the pulley, as shown.
Particle P is released from rest with the string taut and slides along the table.
Assuming P has not reached the pulley, find the tension in the string during the motion.

[8 marks]

Question 3



Two particles P and Q, of masses 3 kg and 2 kg respectively, are attached to the ends of a light inextensible string. P lies on a rough horizontal table. The string passes over a small smooth pulley fixed on the edge of the table. Q hangs freely below the pulley.
The coefficient of friction between P and the table is μ.
The particles are released from rest with the string taut.
Immediately after release, P accelerates at a rate of 0.5 m s^{-2}

(i) Find the tension in the string immediately after the particles begin to move.

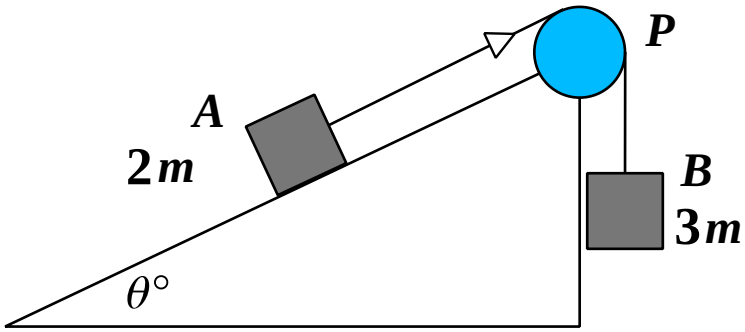
[3 marks]

(ii) Show that $\mu = 0.582$, correct to two significant figures.

[3 marks]

Question 4

A-Level Examination Question from June 2019, Applied, Mechanics, Q3



Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string. Initially A is held at rest on a fixed rough plane.

The plane is inclined at an angle θ to the horizontal ground where $tan\,\theta = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.
The part of the string from A to P is parallel to a line of greatest slope of the plane.
Block B hangs freely below P, as shown.

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.
The tension in the string immediately after the blocks are released is T.
The blocks are modelled as particles and the string is modelled as being inextensible.

Show that,

$$T = \frac{12mg}{5}$$

[8 marks]