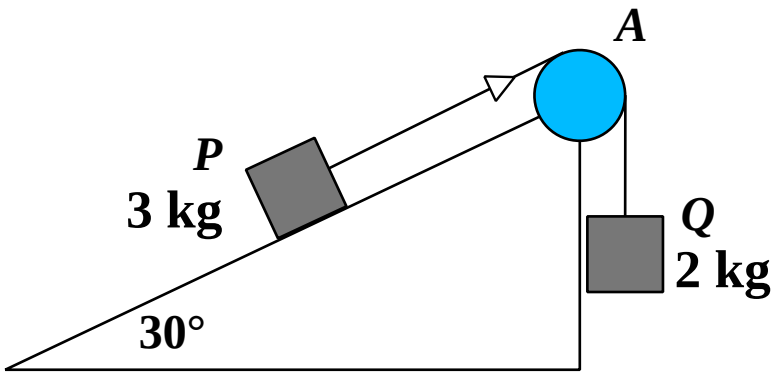
Lesson 6

A-Level Applied Mathematics
Mechanics : Dynamics II : Year 2

6.1 The Slippery Slope

Example



Two particles *P* and *Q* of masses 3 kg and 2 kg respectively are connected by a light inextensible string. Initially *P* is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth pulley *A* fixed at the top of the plane. The part of the string from *P* to *A* is parallel to a line of greatest slope of the plane. The particle *Q* hangs freely below *A*. The system is released from rest with the string taut.

(i) State, with justification, if the 3 kg mass slide up or down the slope.

[2 marks]

(ii) Write down an equation of motion for *P* and an equation of motion for *Q*.

[4 marks]

(iii) Hence show that the acceleration of *Q* is 0.98 m s^{-2}

[2 marks]

(iv) Find the tension in the string.

[2 marks]

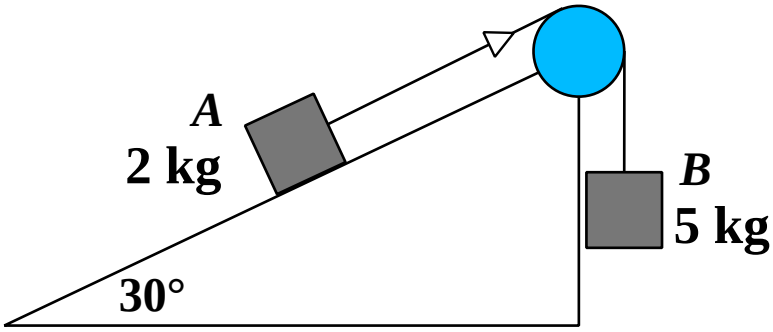
(v) Calculate the magnitude of the force exerted on the pulley by the string.

[2 marks]

6.2 Exercise

Question 1

One end of a light inextensible string is attached to a block *A* of mass 2 kg. The block *A* is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle of 30°. The string lies along the line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a block *B* of mass 5 kg.



The system is released from rest.

By modelling the blocks as particles and ignoring air resistance,

(a) **(i)** show that the acceleration of the block *B* is,

$$a_B = \frac{4}{7} g$$

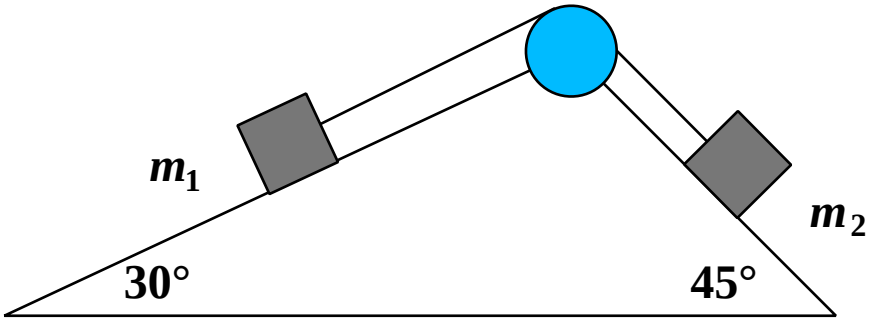
(ii) find the tension in the string

(b) State how you have used the fact that the string is inextensible.

(c) Calculate the magnitude of the force exerted on the pulley by the string.

Question 2

Two particles of masses m_1 and m_2 are connected by a light inextensible string that passes over a smooth pulley. The particles are released from rest on smooth slopes angled at 30° and 45° to the horizontal, as shown.



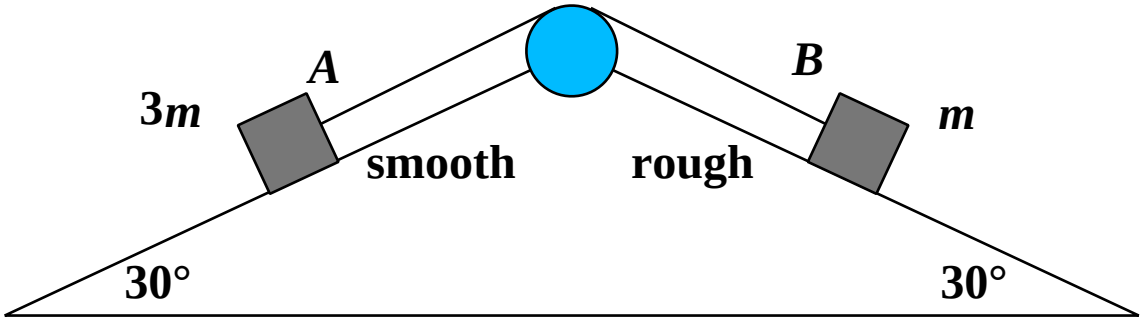
Given that m_2 is accelerating down the 45° slope at 0.5 m s^{-2}, show that,

$$\frac{m_1}{m_2} = \frac{g\sqrt{2} - 1}{1 + g}$$

[6 marks]

Question 3

A fixed wedge has two plane faces, each inclined at 30° to the horizontal. Two particles *A* and *B*, of mass 3*m* and *m* respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a small smooth light pulley fixed at the top of the wedge. The face on which *A* moves is smooth. The face on which *B* moves is rough. The coefficient of friction between *B* and this face is μ. Particle *A* is held at rest with the string taut. The string lies in the same vertical plane as lines of greatest slope on each plane face of the wedge, as shown.



The particles are released from rest and start to move. Particle *A* moves downwards and *B* moves upwards. The accelerations of *A* and *B* each have magnitude 0.1g

(i) By considering the motion of *A*, find, in terms of *m* and *g*, the tension in the string.

[3 marks]

(ii) By considering the motion of *B*, find the value of μ.

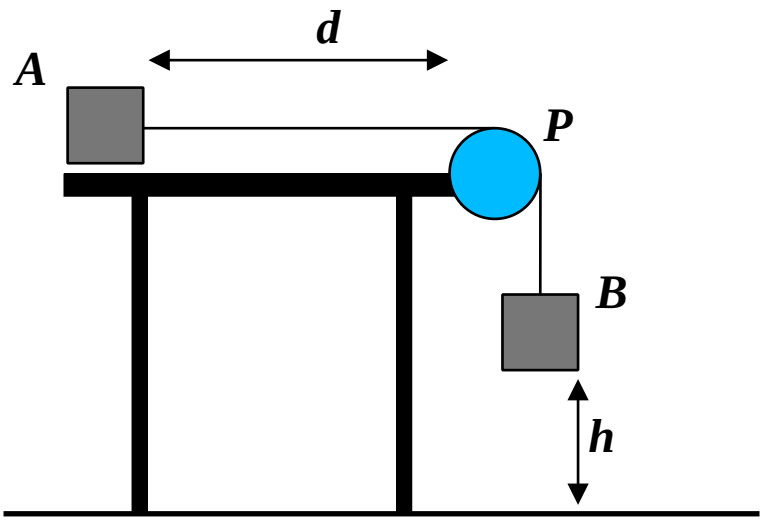
[8 marks]

(iii) Find the resultant force exerted by the string on the pulley, giving its magnitude and direction.

[3 marks]

Question 4

M1 Examination Question from June 2017, Q8



Two particles, A and B, have masses $2m$ and m respectively.
The particles are attached to the ends of a light inextensible string.
Particle A is held at rest on a fixed rough horizontal table at a distance d from a small smooth light pulley which is fixed at the edge of the table at the point P.
The coefficient of friction between A and the table is μ, where $\mu < 0.5$
The string is parallel to the table from A to P and passes over the pulley.
Particle B hangs freely at rest vertically below P with the string taut and at a height h, ($h < d$), above a horizontal floor, as shown.
Particle A is released from rest with the string taut and slides along the table.

(a) **(i)** Write down an equation of motion for A

[2 marks]

(ii) Write down an equation of motion for B

[2 marks]

(b) Hence show that, until B hits the floor, the acceleration of A is

$$a_A = \frac{g}{3}(1 - 2\mu)$$

[3 marks]

(c) Find, in terms of g, h and μ, the speed of A at the instant when B hits the floor.

[2 marks]

After B hits the floor, A continues to slide along the table.

Given that $\mu = \frac{1}{3}$ and that A comes to rest at P,

(d) find d in terms of h.

[5 marks]

(e) Describe what would happen if $\mu = 0.5$

[1 mark]