Lesson 2

2.1 A Crucial Result

When thought about in the right way, the following result is obvious. None-the-less it is a most useful result to have firmly in mind when working on problems involving a hypothesis test.

$$P(X \geqslant k) = 1 - P(X \leqslant k - 1)$$

Proof:

In the following diagram,

the red represents $P(X \geqslant k)$

the green represents 1

the lilac represents $P(X \leqslant k - 1)$

from which the claimed result follows.

x	$P(X = x)$			
0	$P(X = 0)$			
1	$P(X = 1)$			
2	$P(X = 2)$			
3	$P(X = 3)$			
...	...			
$k - 1$	$P(X = k - 1)$			
k	$P(X = k)$			
$k + 1$	$P(X = k + 1)$			
...	...			
n	$P(X = n)$			

The result is useful because the printed tables of cumulative binomial probabilities are only of the form $P(X \leqslant a)$ but a question may ask for $P(X \geqslant b)$. Some calculators (but not all) offer both options.

2.2 An Example Using The Crucial Result

Given that $X \sim B(25, 0.4)$, determine $P(X \geqslant 9)$

2.3 "Spot On™" Example

The standard skin cream treatment for a particular medical skin condition has 0.8 probability of success.

Dr Spot has undertaken research in this area and has produced a new skin cream, "Spot On™", which has been successful with 28 out of 30 patients in a medical trial. Dr Spot claims that the new cream is a 'medical advance'; an improvement on the standard skin cream treatment.

Test, at the 5 % significance level, the claim made by Dr Spot.

2.4 Exercise

Question 1

A single observation, x, is taken from a binomial distribution $B(40, p)$ where p is thought to be 0.25

(**i**)　　What is the expected value of x ?

(**ii**)　　The value of x obtained is higher than expected, $x = 15$.
　　　　Use this observation to test $H_0 : p = 0.25$ against $H_1 : p > 0.25$
　　　　Use a 5 % significance level.

Question 2

A single digit random number generator, generates integers between 0 and 9
It is suspected of being biased in favour of the number 0
It is about to generate 40 random digits.

(**i**)　　How many 0s are expected, assuming the random number generator
　　　　is not biased ?

(**ii**)　　Set up a hypothesis test, by clearly stating the null hypothesis and the
　　　　alternative hypothesis.

(**iii**)　　Here are the 40 numbers generated;

6	1	0	8	4	1	0	0	7
3	0	3	9	5	9	7	9	6
8	8	0	7	3	7	2	6	0
4	8	9	1	3	4	8	1	9
2	0	6	0					

　　　　Is there statistical evidence at the 5 % significance level, to support the
　　　　suspicion ?

Question 3

A psychologist is doing research in the colour preferences of five year old girls.
In one experiment he offers each of 20 girls a choice from three otherwise
identical balls; one pink, one yellow and one blue.
Ten of the girls choose the pink.
He deduces that, of these three colours, five year old girls have a preference
for pink. Is this conclusion justified ?

In your answer clearly state the distribution used along with the null and the
alternative hypothesis employed and the significance level used.

**Question 4**

A dice is rolled 30 times.

(**i**) How many rolls of a six are expected ?

(**ii**) What is the fewest number of rolls of a six needed to persuade you,
at the 5 % significance level, that the dice is biased in favour of a six ?

Take care with 'the crucial result' in giving your final answer.

Question 5

A spinner with eight identical octagonal numbered sectors (as shown) is to be spun 48 times. It is suspected of being biased in favour of the number 3 sector.
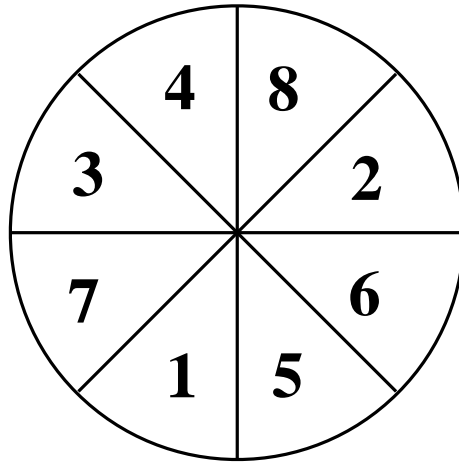
(i)   What is the expected number of 3s, assuming the spinner is not biased ?

(ii)   What is the fewest number of 3s needed to persuade you, at the 5 % significance level, that the spinner is biased in favour of a 3 ?
Clearly state the null and alternative hypothesis used.

The *critical region* is the region of the probability distribution which, if the test statistic falls within it, would cause the null hypothesis to be rejected.

(iii)   State the critical region for question 5

The *critical value* is the first value to fall inside of the critical region.

(iv)   State the critical value for question 5

(v)   When the spinner is spun, sure enough, more 3s than the expected number are obtained. In fact, 15 of the 48 spins resulted in a 3.
Without further calculation, does this observation cause the null hypothesis to be rejected or not ?

Question 6

A random variable, X, has a distribution $X \sim B(30, 0.35)$.

With $H_0 : p = 0.35$ against $H_1 : p > 0.35$ and using a 5 % level of significance, find the critical region of this test.

Question 7

Given a binomial distribution $X \sim B(n, p)$, explain what is meant by;

(i)   A hypothesis test

(ii)   A null hypothesis

(iii)   A critical value

(iv)   An acceptance region

(v)   A 5 % significance level in a one tailed test