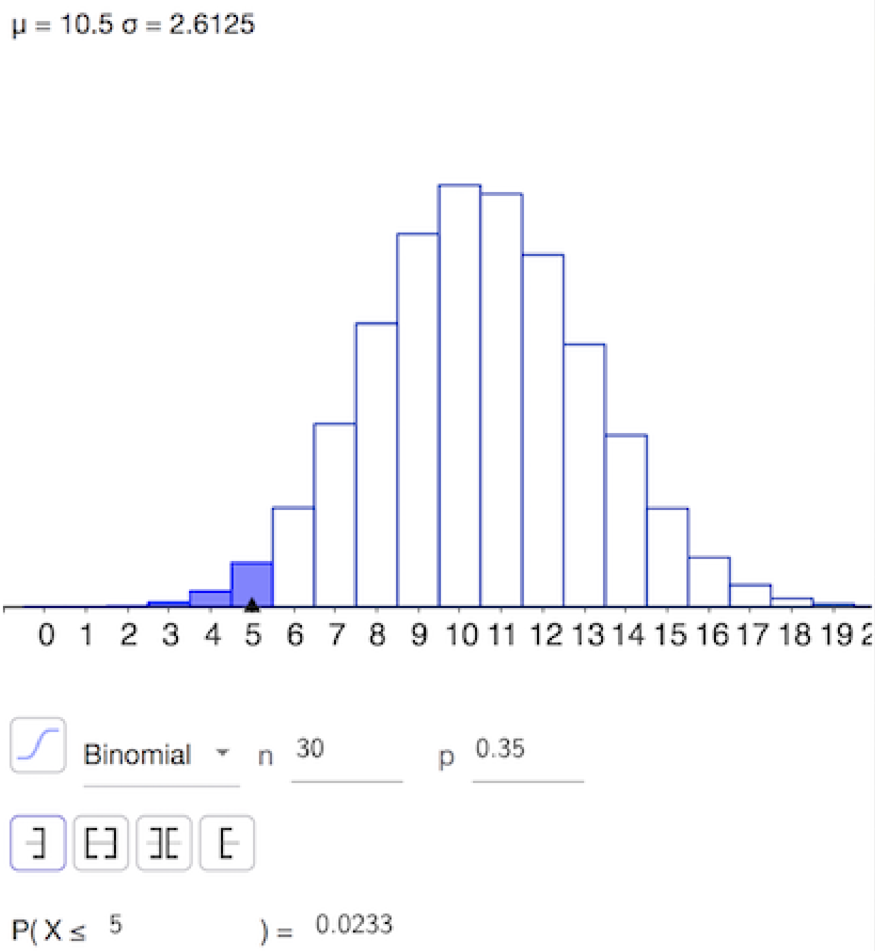
4.1 The Two Tailed Test

We are about to look at an example of a two-tailed hypothesis test. One way to spot that a hypothesis test is two-tailed is if the alternative hypothesis features a "is not equal" sign, like this; $H_1 : p \neq 0.35$

◇ For a two-tailed test, halve the significance level at each tail.

Our example is going to request that a 5 % level of significance be used which means that we are looking at where the critical region of the low tail is about 2.5 %, and the critical region of the high tail is about 2.5 %.

To avoid critical regions that have an actual significance level well above or below the target of 5 %, the example will specify that the probability of rejecting either tail should be ***as close as possible*** to 2.5 %.



k	P(X = k)
0	0
1	0
2	0.0003
3	0.0015
4	0.0056
5	0.0157
6	0.0353
7	0.0652
8	0.1009
9	0.1328
10	0.1502
11	0.1471
12	0.1254
13	0.0935
14	0.0611
15	0.0351
16	0.0177
17	0.0079
18	0.0031
19	0.001
20	0.0003
21	0.0001

The screenshot shows the distribution $X \sim B(n, p)$ with $n = 30$ and $p = 0.35$ and the low tail has been shaded such that the amount shaded is as close to 0.025 as possible. What is shaded is $P(X \leqslant 5) \leqslant 0.0233$ which is 0.0017 away from 0.025. If the $x = 6$ bar was shaded too we'd have $P(X \leqslant 6\ ) \leqslant 0.0586$ which jumps to being way to much !

Now we have a feeling for what is going on, let's do the example.

4.2 Example

Consider a single observation, x, taken from $X \sim B(30, p)$.

This observation is used to test $H_0 : p = 0.35$ against $H_1 : p \neq 0.35$

(i) Explain why this is a two tailed test.

(ii) Using a 5 % level of significance, find the two critical region for this test. The probability of rejecting either tail should be ***as close as possible*** to 2.5 %.

(iii) Why has the critical region is selected in part **(ii)** has been modified to be "***as close as possible*** to 2.5 %" rather than looking for the first value of x a probability below 0.025 and the first above 0.975 ?

(iv) State the actual significance level of this test.

The actual value of X obtained is $x = 4$

(v) State a conclusion that can be drawn based on this value giving a reason for your answer.

4.3 In An Examination

In examinations, if you are short of time, in a question about a two tailed test, you can sometimes get away with only working out the critical region at the tail of interest.

Do this as follows for $X \sim B(n, p)$;

- Work out the expected outcome which is simply np
- If the observed value, x, is lower than this expected outcome, only work out the low tail critical region.
- If the observed value, x, is higher than this expected outcome, only work out the high tail critical region.

In most examination questions it will be obvious which tail you should test.
If a two-tailed examination question asks for the actual significance level, you'll need to tackle both ends to get the two probabilities that need adding together.

4.4 Exercise

Question 1

The national proportion of people experiencing complications after having a particular operation in hospitals is 20 %.

A hospital decides to take a sample of size 20 from their records.

(i) Find critical regions, at the 5 % level of significance, to test whether or not their proportion of complications differs from the national proportion.

The probability in each tail should be as close to 2.5 % as possible.

[5 marks]

(ii) State the actual significance level of the test.

[1 mark]

The hospital finds that 8 of their 20 patients experience complications.

(iii) Comment on this finding in the light of your critical regions

[2 marks]

Question 2

The manager of ***Phones 2 Go*** thinks that the probability is 0.2 of a customer buying an iPhone. To test whether this hypothesis is true the manager decides to record the make of mobile phone bought by a random sample of 50 ***Phones 2 Go*** customers who bought a mobile phone.

(i) Find the two critical regions that would enable the manager to test whether or not there is evidence that the probability is different from 0.2. The probability of each tail should be as close to 2.5 % as possible.

[4 marks]

(ii) Write down the actual significance level of this test.

[2 marks]

From the random sample, 15 customers bought an iPhone.

(iii) Comment on this observation in the light of your critical regions.

[2 marks]

Question 3

The proportion of defective articles in a certain manufacturing process has been found from long experience to be 10 %

A random sample of 50 articles was taken in order to monitor the production.

The number of defective articles was recorded.

(i) Using a 5 % level of significance, find the critical regions for a two-tailed test of the hypothesis that 10 % of articles have a defect.

The probability in each tail should be as near to 2.5 % as possible.

[4 marks]

(ii) State the actual significance level of the above test.

[2 marks]

Another sample, this time of just 20 articles, was taken at a later date.

Four articles were found to be defective.

(iii) Test, at the 10 % significance level, whether or not there is evidence that the proportion of defective articles has increased.

State your hypothesis clearly.

[5 marks]