Grade Grabber 2

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 26

Question 1

A geometric series has first term $sin\ 2\theta$ and common ratio $cos\ 2\theta$

where $0 < \theta < \dfrac{\pi}{4}$

Show that the sum to infinity of this series is $cot\ \theta$

[**4 marks**]

Question 2

Consider the equation;

$$13 \sin \theta + 8 \cos \theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

This question is about solving this equation in two different ways, one approximate, the other exact.

(a)   You are reminded that if θ is small and measured in radians,

- $\sin \theta \approx \theta$ •

- $\cos \theta \approx 1 - \dfrac{\theta^2}{2}$ •

- $\tan \theta \approx \theta$ •

Use the small angle approximations to show that;

$$4 \theta^2 - 13\theta + 3 = 0$$

and hence find a valid approximate solutions to

$$13 \sin \theta + 8 \cos \theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$
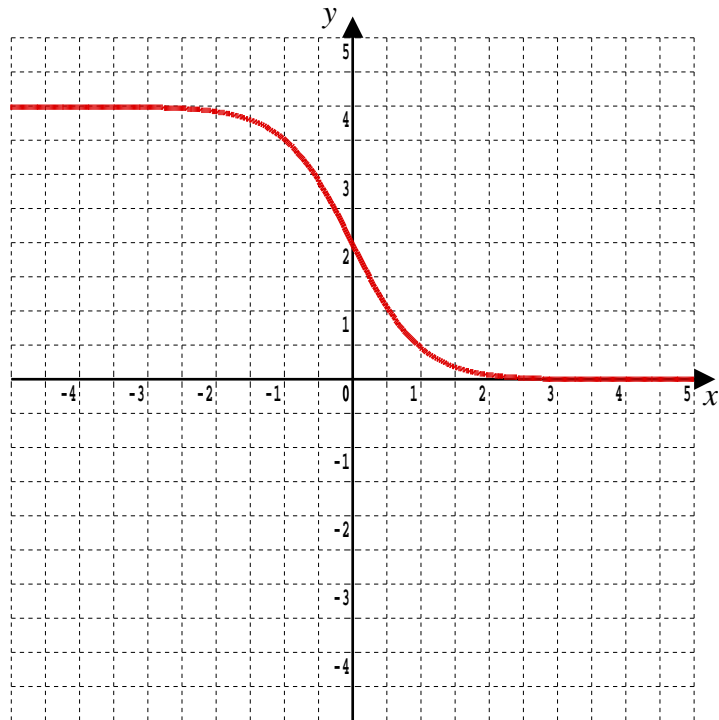
[4 marks]

(b) Assume that

$$13 \sin \theta + 8 \cos \theta = R \sin (\theta + \alpha) \qquad 0 \leqslant \alpha \leqslant \frac{\pi}{2}$$

Determine the value of α and the value of R and hence solve

$$13 \sin \theta + 8 \cos \theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

[4 marks]

Question 3

The graph shows a plot of the curve with equation

$$y = \frac{4}{e^{2x} + 1} \qquad x \in R$$

The area of a finite region, bounded by the curve, the y-axis, the x-axis, and the line with equation $x = 0.5$ is of interest.

(**i**)   On the diagram, shade in the area of interest.

[1 mark]

(**ii**)   Use the substitution $u = e^{2x}$ to show that the area of interest can be given by

$$\int_{a}^{b} \frac{2}{u\,(u + 1)}\, du$$

where a and b are constants to be found

(**iii**) Hence use algebraic integration to show that the exact area
of the region of interest is

$$Area = ln\left(\frac{2e}{e + 1}\right)^2$$

Question 4

One root of the following equation is 2;

$$x^3 + kx + 4 = 0$$

(**i**)　　Find the value of k

[1 mark]

(**ii**)　　Find the exact values of the other two roots.

[4 marks]