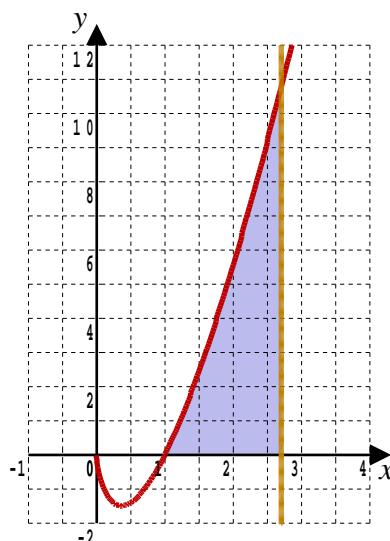


# Grade Grabber 3

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 30

## Question 1



The graph is of the curve  $y = 4x \ln x$ , and the vertical line  $x = e$

Find the exact area bounded by the curve, the  $x$ -axis, and the lines,  $x = 1$  and  $x = e$

[ 6 marks ]

**Question 2**

Prove that

$$\sum_{n=0}^{\infty} x^{2n} = \frac{1}{1 - x^2}$$

[ 4 marks ]

**Question 3**

A curve has parametric equations

$$x = 7 - \frac{3}{4t} \quad y = 8t + 1$$

Show that the Cartesian equation of the curve can be expressed in the form

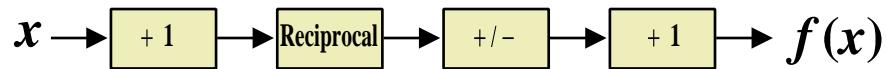
$$y = \frac{a - x}{b - x} \quad x \neq c$$

where  $a, b$  and  $c$  are constants to be found

[ 7 marks ]

**Question 4**

The flowchart describes a function  $f$



(i) Show that the function described can be written as

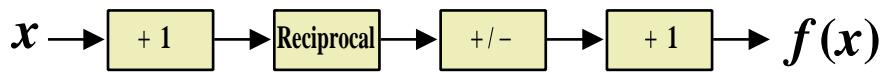
$$f(x) = \frac{x}{x+1} \quad x \neq -1$$

[ 3 marks ]

(ii) Use algebraic manipulation to find  $f^{-1}(x)$

[ 4 marks ]

The task now is to use the flowchart of  $f$  to obtain your part (ii) answer.



- ( iii ) Explain your strategy, drawing an appropriately modified flowchart, and so verify your part (ii) answer from this alternative method.

[ 6 marks ]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com