A-Level Pure Mathematics
Year 1, Year 2 Revision

Grade Grabber 5

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

The gradient of a curve at any point (x, y) on the curve is inversely proportional to the product of x and y. The curve passes through the point (1, 1) and at this point the gradient of the curve is 7.

Form a differential equation in x and y and solve this equation to expess y^2 in terms of x

[7 marks]

Question 2

Relative to a fixed origin *O*, the points *P* and *Q* are such that

$$\overrightarrow{OP} = \begin{pmatrix} 7 \\ 12 \\ -1 \end{pmatrix} \qquad \overrightarrow{OQ} = \begin{pmatrix} 11 \\ 2 \\ a \end{pmatrix}$$

where *p* is a constant, and the points *R* and *S* are such that

$$\overrightarrow{QR} = \begin{pmatrix} 3 \\ -16 \\ 12 \end{pmatrix} \qquad \overrightarrow{PS} = \begin{pmatrix} 1 \\ -11 \\ 16 \end{pmatrix}$$

(i) Find the position vector of the point *S*

[2 marks]

(ii) Find $\overrightarrow{OR}$ in term of a

[2 marks]

(iii) Find $\overrightarrow{SR}$ in term of a

[2 marks]

(iv) Find $\overrightarrow{PQ}$ in term of a

[2 marks]

(v) Given that $\overrightarrow{SR}$ is parallel to $\overrightarrow{PQ}$, find the value of a

[3 marks]

Question 3

Two trigonometry identities involving $cos^2\theta$ and $sin^2\theta$ are;

$$cos^2\theta + sin^2\theta = p$$

$$cos^2\theta - sin^2\theta = q$$

where p is a constant and q is a trigonometric function in θ

(i) Write out the two identities with p and q replaced appropriately

[2 marks]

(ii) Treating the two identities as simultaneous equations, combine them by addition to obtain an expression for $cos^2\theta$

[2 mark]

(iii) Hence, or otherwise, evaluate

$$\int_0^{\frac{\pi}{2}} cos^2\theta \ d\theta$$

[3 marks]

Question 4
Given that

$$x = -3\,cot\,y \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integer constants to be found

[5 marks]