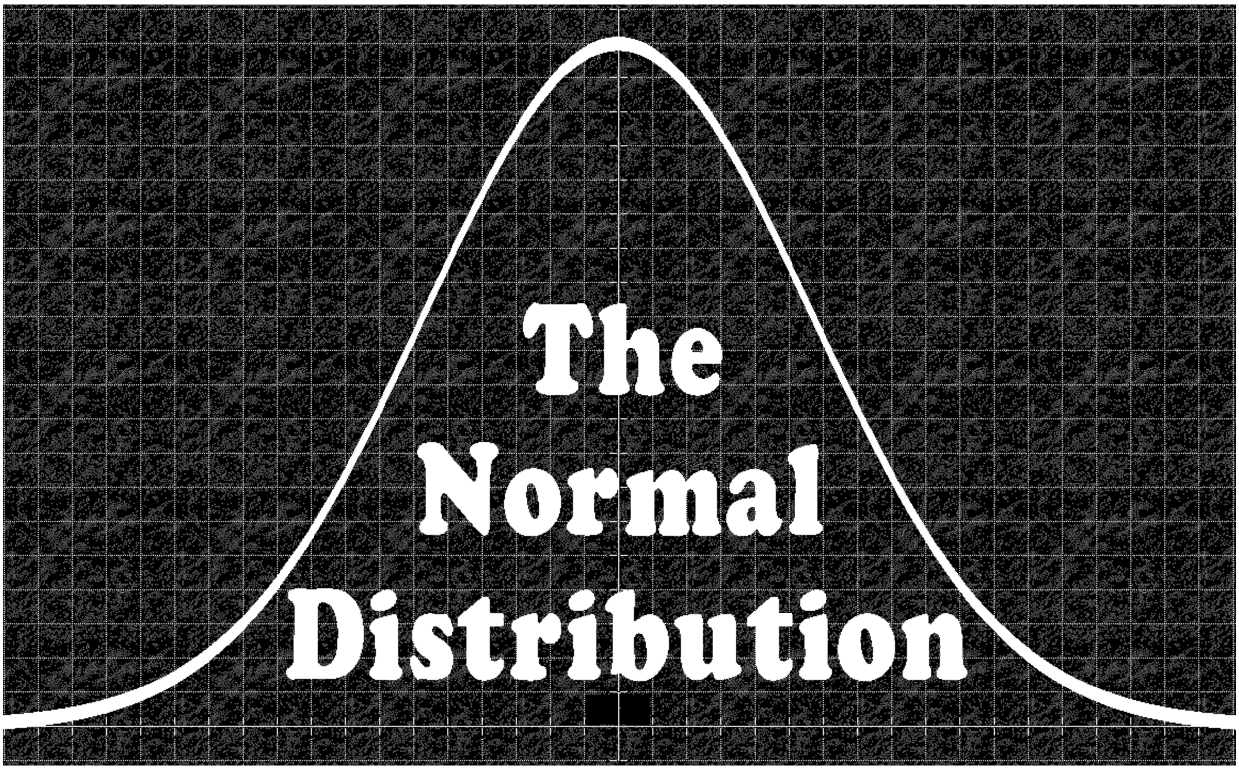
Year 2

~ Statistics ~

T H E
N O R M A L
D I S T R I B U T I O N



Lesson 1

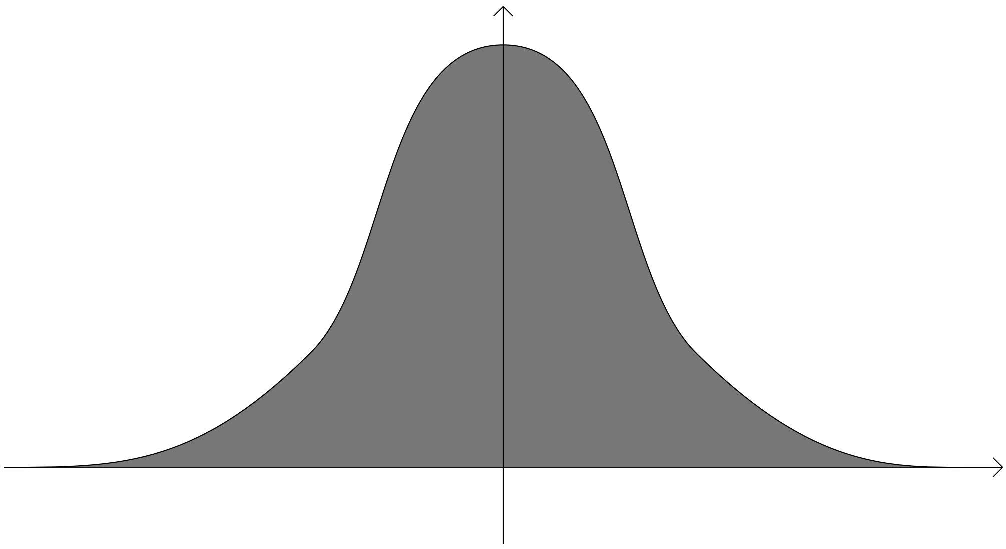
1.1 Getting A Feel For The Normal Distribution

Consider the following fact;

The heights of adult males in the UK are normally distributed with mean 178 cm and standard deviation 18 cm.

This single sentence tells us much about adult males in the UK.

Firstly to describe a continuous variable as ***normal*** means that it's distribution is a symmetrical 'bell-shaped' curve



This curve has equation

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}$$

The total area under the curve, shown shaded, is 1.

This is so that ***area represents probability***.

“Areas under curves” suggests “integration”. Alas, $f(x)$ is not integrable.

In practice, an areas sought will be between limits.

So, having to work numerically rather than algebraically is not a problem;

Numerical methods of integrating to find a specified area under this curve are programmed into your statistics calculator and are easily extracted, when required.

By symmetry, mean = median = mode.

All are 178 cm for our example involving the heights of adult males.

☞ Mark the 178 cm on the above curve.

1.2 Notation

Any data that is ***binomially*** distributed can have the distribution captured using two numbers, *n* the number of trials, and *p* the probability of success.
The binomial distribution was thus a two parameter distribution,

$$X \sim B\ (n,\ p)$$

The normal distribution is also a two parameter distribution. Any data that is ***normally*** distributed can have the distribution captured using two numbers, μ the mean, and σ the standard deviation.

$$X \sim N\ (\mu,\ \sigma^2)$$

σ^2 is called the ***variance***

Thus, for our example...
The heights of adult males in the UK are normally distributed with mean 178 cm and standard deviation 18 cm.
... we could write that

$$X \sim N\ (178,\ 18^2) \qquad or \qquad X \sim N\ (178,\ 324)$$

1.3 Using The Statistics Calculator

Due the the symmetry of the normal distribution, half the area is above the mean and half is below.

$$i.e. \qquad P(X \leqslant \mu) = 0.5$$

When using the statistics calculator to get this result we have to put in numbers and the calculator needs both a lower and an upper limit.
We'd like to ask the calculator to work out this,

$$P(-\infty < X \leqslant \mu)$$

but will have to use a specific numerical example.

So, for our 'adult males' example where μ = 178 cm and σ = 18 cm and -∞ ≈ - 2
(We approximate - ∞ with “ten standard deviations below the mean”)
let's ask the calculator to work out;

$$P(-2 \leqslant X \leqslant 178)$$

Buttons for Casio fx-991 EX Classwiz

On : Menu : 7 : 2 : - 2 : = : 178 : = : 18 : = : 178 : = : =

to get an answer of 0.5

1.4 Exercise

Question 1

To verify that the area above the mean is 0.5 I'd like to ask the statistics calculator to work out

$$P\ (\mu \leqslant X < \infty)$$

(i) Using the 'adult males' example, write down what I will actually have to ask the calculator.

(ii) Verify that you get the expected answer using your calculator.

Question 2

To verify that the area under the normal distribution curve is 1, I'd like to ask the statistics calculator to work out

$$P\ (-\infty < X < \infty)$$

(i) Using the 'adult males' example, write down what I will actually have to ask the calculator.

(ii) Verify that you get the expected answer using your calculator.

Question 3

The probability of being within one standard deviation of the mean is always about 68% regardless of the specific example being looked at.

(i) Use the 'adult males' example to write down what you will ask the calculator to find the probability of being within one standard deviation of the mean.

(ii) Find the percentage probability of being within one standard deviation of the mean correct to five decimal places.

Question 4
The percentage probability of being within two standard deviations of the mean is always the same regardless of the specific example being looked at.

(i) Use the 'adult males' example to write down what you will ask the calculator to find the probability of being within two standard deviations of the mean.

(ii) Find the percentage probability of being within two standard deviations of the mean correct to five decimal places.

(iii) Given that the probability of being within two standard deviations of the mean does not depend upon a specific example, we could let $\mu = \sigma = 1$. Write down what you will ask the calculator if $\mu = \sigma = 1$ to find the probability of being within two standard deviations of the mean and verify this gives your part **(ii)** percentage answer when typed into your calculator.

Question 5
Find the percentage probability of being within three standard deviation of the mean correct to five decimal places.

Question 6
Find as a percentage to 5 decimal places,

$$P\ (\mu - 1.5\sigma \leqslant X \leqslant \mu + \sigma)$$

Question 7
Find as a percentage to 5 decimal places,

$$P\ (X \leqslant \mu + 1.2\sigma)$$

Question 8

The diameters of a rivet produced by a particular machine, X mm, is modelled as

$$X \sim N\ (9,\ 0.2^2)$$

(i) Find $P\ (X > 9)$

(ii) How many standard deviations above the mean is 9.6 mm ?

(iii) Find $P\ (9 \leqslant X \leqslant 9.6)$

(iv) Find $P\ (X > 9.3)$

Question 9

The lengths of a bolt produced by a particular machine, X mm, is modelled as

$$X \sim N\ (36,\ 0.25)$$

(i) Find $P\ (X < 36)$

(ii) How many standard deviations below the mean is 35.25 mm ?

(iii) Find $P\ (35.25 \leqslant X \leqslant 36)$

(iv) Find $P\ (X \leqslant 35.25)$

Question 10

$$X \sim N\ (30,\ 4^2)$$

Find **(i)** $P\ (X < 33)$

(ii) $P\ (X \geqslant 24)$

(iii) $P\ (33.5 \leqslant X \leqslant 38.2)$

(iv) $P\ (X < 27 \text{ or } X > 32)$