Lesson 3

3.1 Coding and The Normal Distribution

A standard mathematical technique is to take an awkward problem and move it to a place where it is more easily solved. Having solved the problem in the 'easier place' the answer then needs to be moved back to the original setting.

One way of moving a problem to an 'easier place' is to use coding.
Given any normal distribution, the following coding is frequently used to transform the given normal distribution into a normal distribution with mean 0 and standard deviation 1;

$$Z = \frac{X - \mu}{\sigma}$$

In the original setting we had ***x*-values** along the *x*-axis.
After the coding has been applied we have ***z*-values** along the *z*-axis.

Statement :
What makes this particular coding so useful is that the *z*-values correspond to the number of standard deviations away from the mean.
Furthermore the coded data has mean of 0 and standard deviation of 1.

Proof :
Let x be m standard deviations away from the mean, i.e. $x = m\,\sigma + \mu$
Then, under the coding, μ moves to,

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{\mu - \mu}{\sigma}$$

$$z = 0$$

And under the coding a point x, (i.e. $m\sigma + \mu$) moves to,

$$z = \frac{m\sigma + \mu - \mu}{\sigma}$$

$$= \frac{m\sigma}{\sigma}$$

$$= m$$

And so the numbers on the *z*-axis are the m numbers which were the number of standard deviations away from the mean, as claimed.

In particular, 1σ goes to 1
i.e. the standard deviation of the coded data is 1. □

3.2 When To Use Coding In A Normal Distribution Question

Whenever a question asks for an unknown mean, μ, or standard deviation, σ, coding will need to be used. It's always the standard coding;

$$Z = \frac{X - \mu}{\sigma}$$

3.3 Example

Consider the random variable $X \sim N(\mu, 16)$

Given that $P(X < 11) = 0.2$, find the value of μ

3.4 Example

Consider the random variable $X \sim N(24, \sigma^2)$

Given that $P(X > 31) = 0.15$, find the value of σ

3.5 Exercise

Question 1

Consider the random variable $X \sim N\,(18, \sigma^2)$

Given that $P\,(X < 15) = 0.01$, find the value of σ

Question 2

Consider the random variable $X \sim N\,(\mu, 9^2)$

Given that $P\,(X > 42) = 0.25$, find the value of μ

Question 3

S1 Examination Question from January 2008, Q6

The weights of bags of popcorn are normally distributed with mean of 200 g and 60% of all bags weighing between 190 g and 210 g.

(a) Write down the median weight of the bags of popcorn

[1 mark]

(b) Find the standard deviation of the weights of the bags of popcorn

[5 marks]

A shopkeeper finds that customers will complain if their bag of popcorn weighs less than 180 g

(c) Find the probability that a customer will complain

[3 marks]

Question 4

S1 Examination Question from May 2013, Q6

The weight, in grams, of beans in a tin is normally distributed with mean μ and standard deviation 7.8

Given that 10% of tins contain less than 200 g, find

(a) the value of μ

[3 marks]

(b) the percentage of tins that contain more than 225 g of beans

[3 marks]

The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with mean 205 and standard deviation σ

(c) Given that 98% of tins contain between 200 g and 210 g find the value of σ

[4 marks]

Question 5

S1 Examination Question from January 2009, Q6

The random variable X has a normal distribution with mean 30 and standard deviation 5

(a) Find $P\ (\ X < 39\ )$

[2 marks]

(b) Find the value of d such that $P\ (\ X < d\ ) = 0.1151$

[4 marks]

(c) Find the value of e such that $P\ (\ X > e\ ) = 0.1151$

[2 marks]

(d) Find $P\ (\ d < X < e\ )$

[2 marks]