Lesson 4

The Normal Distribution
A-Level Applied Mathematics : Statistics : Year 2

4.1 Simultaneous Equations

The examination often contains a Normal Distribution question in which two equations have to be formed, each containing the unknown mean, μ, and unknown standard deviation, σ.
These equations are then solved simultaneously, to find μ and σ.

4.2 Example

The time taken, in minutes, by candidates to answer this mathematics question in an examination is assumed to be normally distributed with mean μ and standard deviation σ.
This question can be completed in less than 12 minutes by 75% of candidates.
For 8% of candidates, it takes more than 16 minutes to complete this question.

(i) Show this information on a diagram of the normal distribution.

(ii) Write down the percentage of candidates who take between 12 and 16 minutes to answer this question.

(iii) Form two equations in μ and σ and hence find, to 2 significant figures, the value of μ and the value of σ

4.3 Exercise

Question 1

S1 Examination Question from January 2010, Q7

The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information

[3 marks]

(b) Show that $\mu = 154 + 1.6449\,\sigma$

[3 marks]

(c) Obtain a second equation and hence find the value of μ and the value of σ

[4 marks]

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm

[3 marks]

Question 2

S1 Examination Question from May 2006, Q5

From experience a high-jumper knows that he can clear a height of at least 1.78 m once in 5 attempts. He also knows that he can clear a height of at least 1.65 m on 7 out of 10 attempts.

Assuming that the heights the high-jumper can reach follow a Normal distribution,

(a) Draw a sketch to illustrate the above information

[3 marks]

(b) Find, to 3 decimal places, the mean and the standard deviation of the heights the high-jumper can reach,

[6 marks]

(c) Calculate the probability that he can jump at least 1.74 m

[3 marks]

Question 3

S1 Examination Question from May 2012, Q6

The heights of an adult female population are normally distributed with mean 162 cm and standard deviation 7.5 cm

(a) Find the probability that a randomly chosen adult female is taller than 150 cm

[3 marks]

Sarah is a young girl.
She visits her doctor and is told that she is at the 60th percentile for height.

(b) Assuming that Sarah remains at the 60th percentile, estimate her height as an adult

[3 marks]

The heights of an adult male population are normally distributed with standard deviation 9.0 cm

Given that 90% of adult males are taller than the mean height of adult females,

(c) find the mean height of an adult male

[4 marks]

Question 4

S1 Examination Question from January 2011, Q8

The weight, X grams, of soup put in a tin by machine A is normally distributed with a mean of 160 g and a standard deviation 5 g.

A tin is selected at random.

(a) Find the probability that this tin contains more than 168 g

[3 marks]

The weight stated on the tin is w grams

(b) Find w such that $P (X < w) = 0.01$

[3 marks]

The weight, Y grams, of soup put into a carton by machine B is normally distributed with mean μ grams and standard deviation σ grams

(c) Given that

$$P\ (\ Y\ <\ 160\ )\ =\ 0.99$$

and

$$P\ (\ Y\ >\ 152\ )\ =\ 0.90$$

find the value of μ and the value of σ

[6 marks]