

Lesson 5

The Normal Distribution A-Level Applied Mathematics : Statistics : Year 2

5.1 Approximating A Binomial Distribution

On the face of it, it would seem that the Normal Distribution is a very different mathematically, in comparison to the Binomial Distribution.

- ◇ The Normal distribution is continuous; the Binomial distribution discrete
- ◇ The Normal distribution is symmetrical, the Binomial can be very skewed

However, in certain situations the Normal Distribution can approximate the Binomial distribution very well.

Essentially, n needs to be large and p close to 0.5

There's no definite rule but $n > 50$, with $np > 5$ is worth keeping in mind.

Here is what this course recommends:

If n is large and p is close to 0.5, then the binomial distribution $X \sim B(n, p)$ can be approximated by the normal distribution $N(\mu, \sigma^2)$ where

$$\mu = np$$
$$\sigma = \sqrt{np(1-p)}$$

5.2 Example

A biased coin has a probability of landing 'heads' of 0.56.

Let X be the number of heads obtained in 50 tosses of this coin.

(i) Use your calculator's Binomial CD function to determine $P(X \leq 33)$

(ii) Use an appropriate approximation involving the Normal distribution to determine $P(X \leq 33)$

(iii) How do your two answers compare ?

5.3 Exercise

Question 1

The binomial random variable $X \sim B(150, 0.48)$ is approximated by the normal random variable $Y \sim N(72, 6.12^2)$

- (i) Explain how the mean of 72 and the standard deviation of 6.12 have been calculated.
- (ii) Use the approximation to find $P(X \leq 70)$
Don't forget to apply the appropriate *continuity correction*
- (iii) Use the approximation to find $P(80 \leq X < 90)$
Look carefully at the inequalities; apply appropriate *continuity corrections*

Question 2

Explain why it would not be appropriate to approximate the following Binomial distributions with Normal distributions.

- (i) $X \sim B(20, 0.51)$
- (ii) $X \sim B(300, 0.85)$

Question 3

For a particular type of flower bulb, 55% will produce yellow flowers.

A random sample of 80 bulbs are planted, all of which are expected to bloom.

- (i) Set up and state a suitable binomial distribution, and then use your calculator to find the exact probability that precisely 50 bulbs produce yellow flowers.

- (ii) Set up and state a suitable normal distribution that will approximate you part (i) binomial distribution.

- (iii) Taking care over applying the necessary *continuity approximations* use your part (ii) normal distribution to estimate the probability that precisely 50 bulbs produce yellow flowers.

- (iv) What is the percentage error that has been incurred when using the normal approximation to estimate the probability that precisely 50 bulbs produce yellow flowers ?

Question 4

SI Examination Question from May 2008, Q2

In a large college 58% of students are female and 42% are male.

A random sample of 100 students is chosen from the college.

Using a suitable approximation, find the probability that more than half the sample are female.

[7 marks]

Question 5

The random variable $Y \sim B(300, 0.6)$

- (a) Give two reasons why a normal distribution can be used to approximate Y

[2 marks]

- (b) Find, using the normal approximation, $P(150 < Y \leq 180)$

[4 marks]

- (c) Find the largest value of y such that $P(Y < y) < 0.05$

[3 marks]

Question 6

A drill bit manufacturer claims that 52% of its bits last longer than 40 hours.

A random sample of 30 bits is taken and X last longer than 40 hours.

- (a) Find $P(X < 17)$

[1 mark]

A second random sample of 600 drill bits is taken.

- (b) Using a suitable approximation, find the probability that between 300 and 350 bits last longer than 40 hours.

[3 marks]