

5.1 Approximating A Binomial Distribution

On the face of it, it would seem that the Normal Distribution is a very different mathematically, in comparison to the Binomial Distribution.

- ◊ The Normal distribution is continuous; the Binomial distribution discrete
- ◊ The Normal distribution is symmetrical, the Binomial can be very skewed

However, in certain situations the Normal Distribution can approximate the Binomial distribution very well.

Essentially, n needs to be large and p close to 0.5

There's no definite rule but $n > 50$, with $np > 5$ is worth keeping in mind.

Here is what this course recommends:

If n is large and p is close to 0.5, then the binomial distribution $X \sim B(n, p)$ can be approximated by the normal distribution $N(\mu, \sigma^2)$ where

$$\begin{aligned}\mu &= np \\ \sigma &= \sqrt{np(1-p)}\end{aligned}$$

5.2 Example

A biased coin has a probability of landing 'heads' of 0.56.

Let X be the number of heads obtained in 50 tosses of this coin.

(i) Use your calculator's Binomial CD function to determine $P(X \leq 33)$

(ii) Use an appropriate approximation involving the Normal distribution to determine $P(X \leq 33)$

(iii) How do your two answers compare ?

5.3 Exercise

Question 1

The binomial random variable $X \sim B (150, 0.48)$ is approximated by the normal random variable $Y \sim N (72, 6.12^2)$

(i) Explain how the mean of 72 and the standard deviation of 6.12 have been calculated.

(ii) Use the approximation to find $P (X \leq 70)$
Don't forget to apply the appropriate *continuity correction*

(iii) Use the approximation to find $P (80 \leq X < 90)$
Look carefully at the inequalities; apply appropriate *continuity corrections*

Question 2

Explain why it would not be appropriate to approximate the following Binomial distributions with Normal distributions.

(i) $X \sim B (20, 0.51)$

(ii) $X \sim B (300, 0.85)$

Question 3

For a particular type of flower bulb, 55% will produce yellow flowers.

A random sample of 80 bulbs are planted, all of which are expected to bloom.

(i) Set up and state a suitable binomial distribution, and then use your calculator to find the exact probability that precisely 50 bulbs produce yellow flowers.

(ii) Set up and state a suitable normal distribution that will approximate you part (i) binomial distribution.

(iii) Taking care over applying the necessary *continuity approximations* use your part (ii) normal distribution to estimate the probability that precisely 50 bulbs produce yellow flowers.

(iv) What is the percentage error that has been incurred when using the normal approximation to estimate the probability that precisely 50 bulbs produce yellow flowers ?

Question 4

S1 Examination Question from May 2008, Q2

In a large college 58% of students are female and 42% are male.

A random sample of 100 students is chosen from the college.

Using a suitable approximation, find the probability that more than half the sample are female.

[7 marks]

Question 5

The random variable $Y \sim B(300, 0.6)$

(a) Give two reasons why a normal distribution can be used to approximate Y

[2 marks]

(b) Find, using the normal approximation, $P(150 < Y \leq 180)$

[4 marks]

(c) Find the largest value of y such that $P(Y < y) < 0.05$

[3 marks]

Question 6

A drill bit manufacturer claims that 52% of its bits last longer than 40 hours.

A random sample of 30 bits is taken and X last longer than 40 hours.

(a) Find $P(X < 17)$

[1 mark]

A second random sample of 600 drill bits is taken.

(b) Using a suitable approximation, find the probability that between 300 and 350 bits last longer than 40 hours.

[3 marks]

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