## 6.1 Hypothesis Testing The Mean

Suppose that we have a large population, *X*, that is normally distributed;

$$X \sim N(\mu, \sigma^2)$$

If we take lots of different random samples of size *n* from the population then it turns out that their means are normally distributed.

This normal distribution of the sample mean is related to that of the parent population. In fact;

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \qquad \textit{or equivalently} \qquad \bar{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$$

This gives us a way to test if the claimed mean of the parent population is likely to be true by looking at the mean of a sample and performing a hypothesis test.

## 6.2 Example

The fizzy drink, *Sugar High*, is sold in cans. The amount of *Sugar High* in a can, in millilitres, follows a normal distribution of mean $\mu$ and standard deviation 16. The manufacturer claims that $\mu$ is 330 ml but a consumer group believe this is too high and that the actual mean is lower. This consumer group takes a sample of 25 cans and calculates the mean amount of *Sugar High* per can to be 323 ml. Test, at the 1% level, whether or not there is evidence that the value of $\mu$ is lower than the manufacturer's claim. State your hypotheses clearly.

## 6.3 Exercise

**Question 1**

The popular washing power, *Scrubs Up Nice,* is sold in boxes. The amount of powder in a box has a normal distribution with standard deviation of 30 grams.
The producer of *Scrubs Up Nice* claims that the mean amount of powder per box, $\mu$, is 600 grams. A trading standards inspector has received complaints that the producer is overstating the mean amount of powder per box and he investigates this by taking a random sample of 16 boxes.
He finds that the mean amount of powder per box is 591 grams.
Using a 5% level of significance, and stating your hypotheses clearly, test whether or not there is statistical evidence to uphold the complaints.

**Question 2**

A manufacturer of model railway N Gauge track aims to make the inside track width of each piece of track, on average, 9 mm, with standard deviation 0.75 mm. The track widths of all the pieces of track may be assumed to be normally distributed. As part of quality control, a random sample of 20 pieces of railway track are tested, and the mean width between the rails is found to be 9.3 mm.

Test at the 5% significance level if this test statistic is compatible with the manufacturers intended mean. State your hypotheses clearly.

**HINT :** This is a two-tailed test

**Question 3**
In each part, a random sample of size *n* is taken from a population having a normal distribution with mean $\mu$ and variance $\sigma^2$
Test the hypotheses at the stated levels of significance.

**( i )**

$$H_0 : \mu = 21 \qquad H_1 : \mu \neq 21$$
$$n = 20 \qquad \bar{x} = 21.7 \qquad \sigma = 1.5 \qquad 5\% \; level$$

**( ii )**

$$H_0 : \mu = 100 \qquad H_1 : \mu < 100$$
$$n = 36 \qquad \bar{x} = 98.5 \qquad \sigma = 5.0 \qquad 5\% \; level$$

**( iii )**

$$H_0 : \mu = 5 \qquad H_1 : \mu \neq 5$$

$$n = 25 \qquad \bar{x} = 6.1 \qquad \sigma = 3.0 \qquad 5\% \ level$$

**( iv )**

$$H_0 : \mu = 15 \qquad H_1 : \mu > 15$$

$$n = 40 \qquad \bar{x} = 16.5 \qquad \sigma = 3.5 \qquad 1\% \ level$$

**( v )**

$$H_0 \; : \; \mu \; = \; 50 \qquad H_1 \; : \; \mu \; \neq \; 50$$

$$n \; = \; 60 \qquad \bar{x} \; = \; 48.9 \qquad \sigma \; = \; 4.0 \qquad 1\% \; level$$