7.1 The Critical Region(s) Method of Hypothesis Testing The Mean

An alternative Hypotheses Testing strategy to that looked at in Lesson 6, is to first establish the critical regions for the test, and then consider a test statistic.

We are still working with a large population, *X*, that is normally distributed;

$$X \sim N(\mu, \sigma^2)$$

and we are still considering a random sample of size *n* taken from this population.

Recall that the normal distribution of the sample mean is related to that of the parent population;

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \qquad \textit{or equivalently} \qquad \bar{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$$

7.2 Example

A machine produces high pressure diesel fuel pipes for the automotive industry.
The diameter of the pipes are normally distributed with mean 0.580 cm and standard deviation 0.015 cm
Following a service of the machine it is assumed that the pipes produced continue to have diameters that are normally distributed with the same standard deviation, but it is wished to check if the mean of the diameters has changed.
Consequently a random sample of 50 fuel pipes is taken.

(i) Find at the 1% level, the critical regions for this test.
State your hypotheses clearly.

The mean diameter of the sample of 50 fuel pipes is calculated to be 0.587 cm

(ii) Comment on this observation in light of your part **(i)** critical regions.

7.3 Exercise

Question 1

The IQ scores of a population are normally distributed with a mean of 100 and a standard deviation of 15. A psychologist wishes to test the theory that eating chocolate before sitting an IQ test improves your score. A random sample of 80 people are selected and they are each given an identical bar of chocolate to eat before taking an IQ test

(i) Find, at the 2.5% level, the critical region for this test.
State your hypotheses clearly.

The mean score on the test for the sample of 80 people was 102.5

(ii) Comment on this observation in light of the critical region.

Question 2

In each part, a random sample of size n is taken from a population having a normal distribution with mean μ and variance σ^2

Find the critical region(s) for the test statistic $\bar{X}$ in the following tests

(i)

$$H_0 : \mu = 120 \qquad H_1 : \mu < 120$$
$$n = 30 \qquad \sigma = 2 \qquad 5\% \ level$$

(ii)

$$H_0 : \mu = 12.5 \qquad H_1 : \mu > 12.5$$
$$n = 25 \qquad \sigma = 1.5 \qquad 1\% \ level$$

(iii)

$$H_0 : \mu = 85 \qquad H_1 : \mu < 85$$

$$n = 50 \qquad \sigma = 4 \qquad 10\%\ level$$

(iv)

$$H_0 : \mu = 0 \qquad H_1 : \mu \neq 0$$

$$n = 45 \qquad \sigma = 3 \qquad 5\%\ level$$

(v)

$$H_0 : \mu = -8 \qquad H_1 : \mu \neq -8$$

$$n = 20 \qquad \sigma = 1.2 \qquad 1\%\ level$$

Question 3
The mass of European water voles, M grams, is normally distributed with standard deviation 12 grams.

Given that 2.5% of water voles have a mass greater than 160 grams,

(i) find the mean mass of a European water vole

[3 marks]

Eight water voles are chosen at random

(ii) Find the probability that at least 4 have a mass greater than 150 grams

[3 marks]

European water rats have mass, N grams which is normally distributed with standard deviation 85 grams.

A random sample of 15 water rats is taken and the sample mean mass is found to be 875 grams.

(iii) Stating your hypotheses clearly, and using a 10% level of significance, test whether the mean mass of all water rats is different from 860 grams

[4 marks]