A-Level Applied Mathematics: Mechanics: Year 2

**Projectiles (Kinematics III)** 

#### 2.1 Launch at an angle

## **Example**

A stone is launched from a catapult, held 2 metres above a flat playing field with a speed of  $64 \text{ m s}^{-1}$  and at an angle of  $25^{\circ}$  above the horizontal.

(i) How long does the stone take to reach its maximum height?

[4 marks]

(ii) What is the maximum height reached, above the playing field?

[3 marks]

(iii) How far, horizontally, is the stone then away from its launch position?

[ 1 mark ]

( iv ) Explain why the horizontal range of the stone is more than twice your part ( iii ) answer.

[2 marks]

#### 2.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 40

### **Question 1**

A cricket ball is projected from the origin at a velocity of ( $5\mathbf{i} + 23\mathbf{j}$ ) m s<sup>-1</sup> where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors in the horizontal and upwards vertical directions respectively. Take the acceleration due to gravity to be of magnitude 9.8 m s<sup>-1</sup>.

(i) Find the particle's position vector after 2.5 seconds. Write your answer in the form  $\mathbf{r} = p\mathbf{i} + q\mathbf{j}$  where p and q are numbers the value of each to be found.

[4 marks]

(ii) Find the particle's velocity vector after 2.5 seconds. Write your answer in the form  $\mathbf{v} = r\mathbf{i} + s\mathbf{j}$  where r and s are numbers the value of each to be found.

A golfer hits a golf ball resting on a tee with a velocity of  $52 \text{ m s}^{-1}$  at an angle of  $58^{\circ}$  above the horizontal. Amazingly, the ball lands in the hole; a hole-in-one!



Find the minimum distance the golfer walks to retrieve her ball. Assume that the gold driving range is a sufficiently large flat horizontal surface.

A particle is projected with a speed of 26 m s<sup>-1</sup> with angle of elevation  $\theta$ . If the greatest height reached above the point of projection is 12 metres, find the value of  $\theta$ .

[4 marks]

## **Question 4**

A stone is thrown skywards with speed 28 m s<sup>-1</sup> from a window which is 22 metres above horizontal ground. The stone hits the ground 4 seconds later.

(i) Determine the angle of projection of the stone with respect to the positive x-axis direction.

[ 5 marks ]

(ii) Find the horizontal distance from the window to the point where the stone hits the ground.

[ 1 mark ]

A particle is projected from a point with position vector ( $3\mathbf{i} + 5\mathbf{j}$ ) metres relative to a fixed origin O where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors in the horizontal and upwards vertical directions respectively. The particle passes through the point with position vector ( $15\mathbf{i} + 7\mathbf{j}$ ) metres after 2.5 seconds.

Find the particle's initial velocity vector.

Write your answer in the form  $\mathbf{v} = p\mathbf{i} + q\mathbf{j}$  where p and q are numbers the value of each to be found.

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A particle, which was launched horizontally across a region of flat level groun	ıd,
lands with a speed of 35 m s <sup>-1</sup> at an angle of 54° to the horizontal.	

(i) What was its launch speed?

[2 marks]

(ii) How far has it travelled horizontally?

[ 3 marks ]

(iii) At what height was it launched from?

A particle is projected from a point with speed 21 m s<sup>-1</sup> at an angle of elevation  $\theta$  and moves freely under gravity. When the particle has moved a horizontal distance of x metres, its height above the point of projection is y metres.

(i) Show that,  $y = x \tan \theta - \frac{x^2}{90} (1 + \tan^2 \theta)$ 

[4 marks]

(ii) Given that y = 8.1 when x = 36, find the value of  $\theta$ 

[3 marks]