Lesson 3

A-Level Applied Mathematics : Mechanics : Year 2
Projectiles (Kinematics III)

3.1 Examination Questions on Angled Launch

There will almost certainly be an Angled Launch projectiles question in the examination. In such questions, unless told otherwise, the unit vectors **i** and **j** are in a vertical plane, **i** being horizontal and **j** being vertically upwards.

3.2 The Theory

Horizontally as there is no acceleration, the model is simply;

$$Distance = Speed \times Time$$

Vertically as there is constant acceleration due to gravity of magnitude 9.8 m s^{-2} the *suvat* equations apply;

$$v = u + at \qquad s$$

$$s = vt - \frac{1}{2} a t^2 \qquad u$$

$$s = ut + \frac{1}{2} a t^2 \qquad v$$

$$s = \left(\frac{v + u}{2} \right) t \qquad a$$

$$v^2 = u^2 + 2as \qquad t$$

s = *displacement*

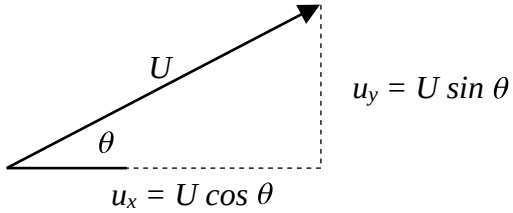
u = *initial velocity*

v = *final velocity*

a = *acceleration (constant)*

t = *time*

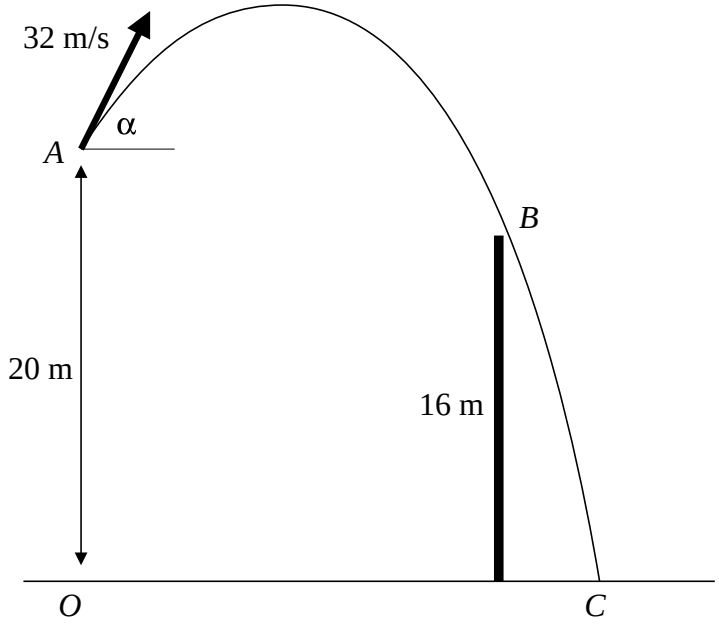
Between Horizontal and Vertical



When a particle is launched with a speed U at an angle θ,

- the compontent of the initial velocity in the x-axis direction is $u_x = U \cos \theta$
- the component of the initial velocity in the y-axis direction is $u_y = U \sin \theta$

3.3 Example



A particle P is projected from a point A with speed 32 m s^{-1} at an angle of elevation α, where $sin\ \alpha = \frac{3}{5}$. The point O is on horizontal ground, with O vertically below A and OA = 20 metres. The particle P moves freely under gravity and passes through a point B which is 16 metres above the ground, before reaching the ground at the point C.

Calculate

(a) The time of flight from A to C

[5 marks]

(b) the distance OC

[3 marks]

(c) the speed of P at B

[4 marks]

(d) the angle that the velocity of P at B makes with the horizontal

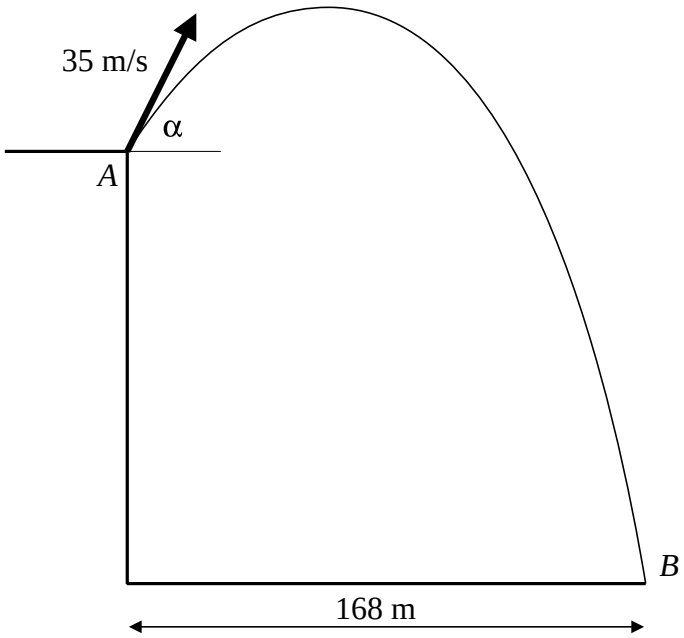
[3 marks]

3.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 39

Question 1



A golf ball *P* is projected with speed 35 m s^{-1} from a point *A* on a cliff above horizontal ground.

The angle of projection is α to the horizontal, where $tan\ \alpha = \frac{4}{3}$

The ball moves freely under gravity and hits the ground at the point *B*.

(a) Find the greatest height of *P* above the level of *A*

[3 marks]

The horizontal distance from A to B is 168 metres.

(b) Find the height of A above the ground.

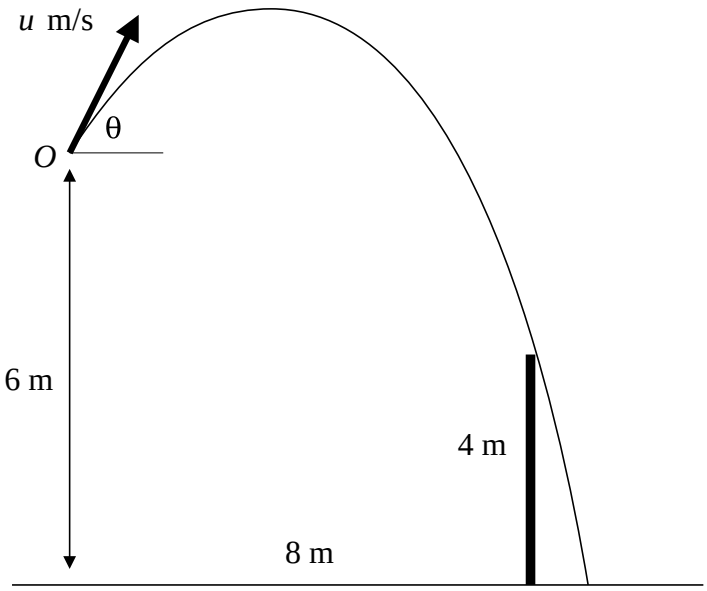
[6 marks]

(c) Find the speed of P as it hits the ground at B

[3 marks]

Question 2

A-Level Examination Question from January 2013, M2, Q6 (Edexcel)



A ball is thrown from a point O, which is 6 metres above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 metres high and 8 metres horizontally away from the vertical through O. The ball passes just above the top of the post 2 seconds after projection. The ball is modelled as a particle.

(a) Show that $tan\ \theta = 2.2$

[5 marks]

(b) Find the value of u

[2 marks]

The ball hits the ground T seconds after projection.

(c) Find the value of T

[3 marks]

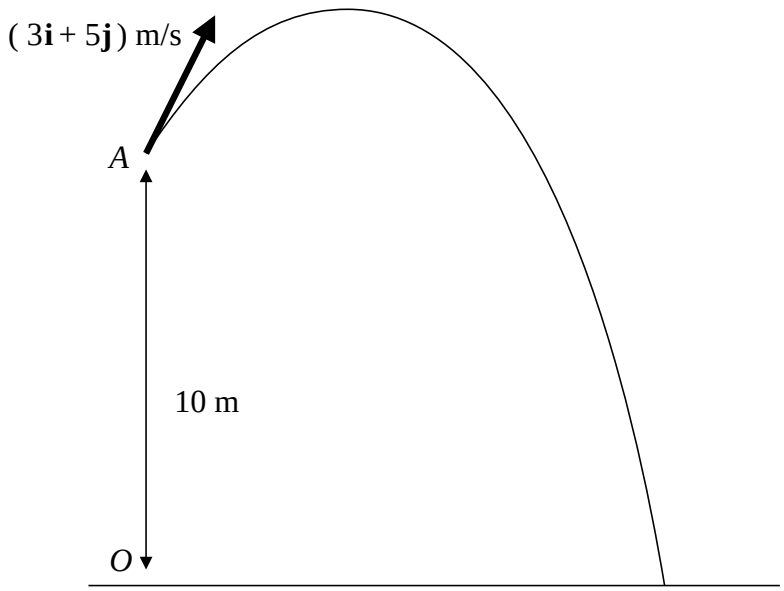
Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(d) Find α

[5 marks]

Question 3

A-Level Examination Question from January 2011, M2, Q6 (Edexcel)



At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}. The particle moves freely under gravity and reaches the ground after T seconds.

(a) For $0 \leqslant t \leqslant T$, show that, with respect to O, the position vector, $\boldsymbol{r}$ metres, of P at time t seconds is given by

$$\boldsymbol{r} = 3t\,\mathbf{i} + \left(10 + 5t - 4.9\,t^2 \right) \mathbf{j}$$

[3 marks]

(b) Find the value of T

[3 marks]

(c) Find the velocity of P at time t seconds $(0 \leqslant t \leqslant T)$

[2 marks]

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(d) Find the time taken for P to move from A to B

[2 marks]

(e) Find the speed of P as it passes through B

[2 marks]