Lesson 4

A-Level Applied Mathematics : Mechanics : Year 2
Projectiles (Kinematics III)

4.1 The Flight Path

A particle is projected from a point with speed u m s^{-1} at an angle of elevation θ. It moves freely under gravity. When the particle has moved a horizontal distance of x metres, its height above the point of projection is y metres.

The Flight Path has equation

$$y = x\tan\theta - \frac{g x^2}{2u^2}\left(1 + \tan^2\theta\right)$$

where g is the magnitude of the acceleration due gravity, 9.8 m s^{-2}

Proof

(*an examination question may ask for this proof*)

$$\text{Horizontally : } u_H = u\cos\theta$$

$$\text{Vertically : } u_V = u\sin\theta$$

For the horizontal motion,

$$Distance = speed \times time$$

$$x = u\cos\theta \times t$$

$$t = \frac{x}{u\cos\theta}$$

For the vertical motion,

$$s = ut + \frac{1}{2}at^2$$

$$y = u \times \sin\theta \times t + \frac{1}{2} \times (-g) \times t^2$$

$$= u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2}g \times \left(\frac{x}{u\cos\theta}\right)^2$$

But $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$

$$\therefore\ y = x\tan\theta - \frac{g x^2}{2u^2}\sec^2\theta$$

But $\sec^2\theta = 1 + \tan^2\theta$

$$\therefore\ y = x\tan\theta - \frac{g x^2}{2u^2}\left(1 + \tan^2\theta\right)$$

□

4.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 35

Question 1

A shot-putter puts a shot from a point *A* at a height of 2 metres above horizontal ground. The shot is projected at an angle of elevation of 45° with a speed of 14 m s^{-1}. By using the flight path equation for a particle moving freely under gravity find, to 3 significant figures, the horizontal distance of the shot from *A* when the shot hits the ground.

[5 marks]

Question 2

A boy throws a stone from a point *P* at the end of a pier.

The point *P* is 15 metres above sea level.

The stone is projected with a speed of 8 m s^{-1} at an angle of elevation of 40°

By using the flight path equation for a particle moving freely under gravity find, to 3 significant figures, the horizontal distance of the stone from *P* when the stone hits a seagull flying 2 metres above sea level.

[5 marks]

Question 3

A projectile is fired with velocity 40 m s^{-1} at an angle of elevation of 30° from a point *A* on horizontal ground. The projectile moves freely under gravity until it reaches the ground at the point *B*.

(i) Find the distance between *A* and *B*

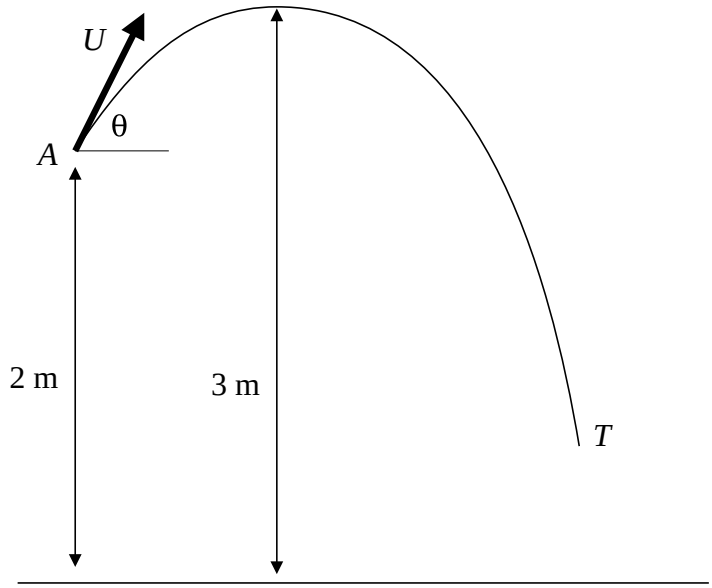
[5 marks]

(ii) What is the speed of the projectile at the first instant when it is 15 metres above the ground ?

[5 marks]

Question 4

A-Level Examination question from June 2018, Q10



A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point *A*, the ball is 2 metres above horizontal ground and is moving with speed *U* at an angle θ above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 metres above the ground. The target is modelled as being the point *T*, as shown. The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that, $U^2 = \dfrac{2g}{sin^2\,\theta}$

[2 marks]

The point T is at a horizontal distance of 20 meters from A and is at a height of 0.75 metres above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle θ

[9 marks]

(c) State one limitation of the model that could affect your answer to part (b)

[1 mark]

(d) Find the time taken for the ball to travel from A to T

[3 marks]