Twenty-One Today

N U M B E R 2

Twenty-One short, sharp questions on all aspects of the Year 1 course
You May Use A Calculator

Question 1

Find

$$\int_4^{25} \sqrt{x}\ dx$$

Question 2

Differentiate $y = 4x^3 + 5x$

Question 3

$$f(x) = \frac{12}{x^2}$$

Determine $f'(-2)$

Question 4

$$f(x) = x^2 - 9 \qquad x \in \mathbb{R}$$

Explain why the function $f(x)$ does NOT have an inverse.

Question 5

$$g(x) = \frac{2x^2 + 9x - 18}{(x + 6)(x + 1)} \qquad x \in \mathbb{R},\ x \neq 1,\ 6$$

(i) Simplify $g(x)$ as far as possible.

(ii) Hence, give an expression for $g^{-1}(x)$ also simplified as far as possible.

Question 6

Sketch the curve of the equation $y = x^2 - 16$, giving the coordinates of any points where it crosses axes.

Question 7

Using Question 6, solve the inequality,

$$x^2 - 16 > 0$$

Question 8

Determine the gradient of the line with equation,

$$2y - 5x - 1 = 0$$

Question 9

The vectors $5\boldsymbol{a} + k\boldsymbol{b}$ and $8\boldsymbol{a} + 56\boldsymbol{b}$ are parallel.
Find the value of k

Question 10

What is the value of the discriminant of $x^2 + 12x + 37 = 0$?

Question 11

The equation $x^2 + 2kx + 4k = 0$, where k is a non-zero integer, has equal roots. Find the value of k.

Question 12

Definition :

A unit vector in the direction of $\boldsymbol{a}$ *is* $\dfrac{\boldsymbol{a}}{|\boldsymbol{a}|}$

Given that $\boldsymbol{a} = 6\boldsymbol{i} + 8\boldsymbol{j}$ find

(i) $|\boldsymbol{a}|$

(ii) a unit vector in the direction of $\boldsymbol{a}$

Question 13

Sketch the curve with equation

$$y = x^2 - 4x + 3$$

Annotate your sketch with the coordinates of the three points where the curve cuts the axes.

Question 14

Beginning "LHS = ", show that

$$\frac{3x^2 + 4\sqrt{x}}{2x} = 1.5x + 2x^{-\frac{1}{2}}$$

Question 15

By considering Question 14, determine $g'(x)$

$$g(x) = \frac{3x^2 + 4\sqrt{x}}{2x}$$

Question 16

Consider the Arithmetic Sequence 8, 1, x, y, ...

Determine the value of x and the value of y.

Question 17

The formula for the n^{th} term of an Arithmetic progression is given in the Exam Formulae book:

$$U_n = a + (n - 1)\,d$$

along with two formula to sum such a series:

$$Sum_{AP} = \frac{1}{2} n\,(a + l) = \frac{n}{2}\{\,2a + (n - 1)\,d\,\}$$

where a is the first term

n is the number of terms

d is the common difference

l is the last term

Use two of these formulae to calculate;

$$4 + 15 + 26 + \ldots + 213$$

Question 18

Determine the value of this series
(which is not in Arithmetic or Geometric Progression)

$$\sum_{1}^{4}\left(m^2 - 1\right)$$

Question 19

The point $(3,\ 5)$ is on the curve $y = x^2 - 2x + c$
State the value of c.

Question 20

Given that $10000\sqrt{10} = 10^k$, find the value of k

Question 21

Solve the simultaneous equations

$$y = 2x - 5$$
$$x^2 + y^2 = 25$$