Twenty-One Today

N U M B E R 4

Twenty-One short, sharp questions on all aspects of the Year 1 course
You may use a calculator

Question 1

Rationalise the denominator of, $\dfrac{\sqrt{2}\ +\ 3}{5\ +\ \sqrt{2}}$

Question 2

Given that, $\int_2^a x^2\,dx\ =\ 111\dfrac{2}{3}$

determine the value of the constant a

Question 3

$$f\,(x)\ =\ 5x^6\ +\ 8x^5\ -\ 13$$

Is $(\,x\ -\ 1\,)$ a factor of $f\,(x)$?

Justify your answer.

Question 4

What is the circumference of the circle with equation,

$$(x + 3)^2 + (y - 8)^2 = 64$$

Write your answer as a multiple of π.

Question 5

Find the coordinates where the line with equation $y = 4x - \frac{7}{2}$ meets the line with equation $y = -\frac{1}{4}x + 5$

Question 6

(i) What is the minimum value of the function $f(x) = (x + 7)^2 + 5$?

(ii) What is the maximum value of the function $g(x) = 8\sin x + 13$?

(iii) Hence state the ***minimum*** value of,

$$\frac{f(x)}{g(x)}$$

Question 7

Write out Pascal's triangle up to the row that runs, 1, 6, 15, 20, 15, 6, 1

Question 8

Given that, in row 6 of Pascal's Triangle,

$$^{n}C_{r} = 20$$

State the value of *n* and the value of *r*

Question 9

Write without any brackets;

$$\left(\frac{3}{x} - \frac{x^2}{2} \right)^4$$

Question 10

A line segment has endpoints $(3,\ 5)$, $(8, -5)$

Find the equation of the perpendicular bisector of this line.

Question 11

Given that $y = 1.6x^4 + 0.9x^2$, what is $\int y\,dx$

Question 12

(i) Which part of $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is termed the discriminant ?

(ii) Suppose that the discriminant is negative.

What can then be deduced about the graph of $y = ax^2 + bx + c$?

Question 13

Give the equation of the curve which passes through the point (1, 13) and which has a gradient equation given by;

$$\frac{dy}{dx} = 40x^3$$

Question 14

A circle has equation

$$x^2 + y^2 - 10x + 4y = 7$$

Determine the centre of the circle and its radius.

Question 15

Solve the inequality,

$$x^2 - 3 \geqslant 1$$

Question 16

Given that

$$128\sqrt{2} = 2^a$$

find the value of a

Question 17

Solve over the interval $0° \leqslant x \leqslant 360°$,

$$tan^2 x + cos^2 x = 1$$

Question 18

(i) Factorise completely,

$$f(x) = x^3 - x$$

(ii) Hence write down the coordinates of the three points where the graph of $f(x)$ crosses the x-axis

Question 19

With the help of your answer to question 18, or otherwise, sketch the graph of,

$$f(x) = x^3 - x$$

Question 20

Find the equation of the tangent to the curve $y = x^3 - x$ from the point $(1, 0)$

Question 21

Find the area bounded by $y = x^3 - x$, the x-axis, $x = -1$ and $x = 1$