Lesson 4

A-Level Pure Mathematics, Year 2
Geometric Progressions

4.1 A Logarithm Surprise

There is a situation that routinely arise in questions about Geometric Progressions that requires an ability to use logarithms.

4.2 Example

Sum the following series which is in geometric progression;

$$3 + 6 + 12 + 24 + \ ... \ + \ 49152$$

Teaching Video : http://www.NumberWonder.co.uk/v9077/4.mp4

[4 marks]

4.3 Exercise

Marks Available: 40

Question 1

What is the first term in the following geometric progression to exceed 1 million ?

2, 6, 18, 54, 162, ...

HINT :

This is about solving $a\,r^{n-1} > 1\,000\,000$

[4 marks]

Question 2

What is the first term in the following geometric progression to exceed 200 ?

0.4, 0.6, 0.9, 1.35, 2.025, ...

[4 marks]

Question 3

Sum the following series which is in geometric progression;

$$19683 + 6561 + \ldots + 1$$

[4 marks]

Question 4

A population of rabbits is increasing at a rate of 35% per annum on a large and uninhabited island with lush vegetation.
At the start of 2011 there were 40 rabbits.
In what year will the rabbit population first exceed 1000 rabbits ?

HINT : Be careful about exactly what this question is asking.

[4 marks]

Question 5

Sum the following series which is in geometric progression;

$$1 - 2 + 4 - 8 + 16 - 32 + ... + 1073741824$$

HINT : To avoid *ln* (-2) and a 'math error'...

$$(-2)^{n-1} = 1073741824$$

$$(-1)^{n-1}\left(2^{n-1}\right) = 1073741824$$

$(-1)^{n-1}$ *must equal* 1 *and n must be odd*

$$\therefore 2^{n-1} = 1073741824$$

[4 marks]

Question 6

C2 Examination Question, May 2006, Q9

A geometric series has first term a and common ratio r

The second term of the series is 4 and the sum to infinity of the series is 25

(a) Show that

$$25r^2 - 25r + 4 = 0$$

[4 marks]

(b) Find the two possible values of r

[2 marks]

(c) Find the corresponding two possible values of a

[2 marks]

(d) Show that the sum, S_n , of the first n terms of the series is given by

$$S_n = 25\left(1 - r^n\right)$$

[1 mark]

Given that r takes the larger of its two possible values,

(e) find the smallest value of n for which S_n exceeds 24

[2 marks]

Question 7

C2 Examination Question, June 2008, Q6

A geometric series has first term 5 and common ratio $\frac{4}{5}$

Calculate

(a) the 20th term of the series, to 3 decimal places

[2 marks]

(b) the sum to infinity of the series

[2 marks]

Given that the sum to k terms of the series is greater than 24.95

(c) show that

$$k > \frac{log\, 0.002}{log\, 0.8}$$

[4 marks]

(d) find the smallest possible value of k

[1 mark]