A Level Further Mathematics

~ Further Statistics 1 ~

T H E
P O I S S O N
D I S T R I B U T I O N

The Normal distribution under certain conditions modelled the Binomial distribution.
Can it also model the Poisson distribution ?

The Poisson Distribution

Lesson 1

The Poisson Distribution : Further Statistics 1

1.1 Introducing The Poisson Distribution

Named after the French mathematician, Siméon Denis Poisson (1781-1840), the Poisson distribution is a discrete probability distribution.
What other discrete probability distribution has this course previously featured ?

☞

The Poisson distribution expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant average rate and independently of the time since the last event.

If $X \sim Po(\lambda)$, then the Poisson distribution is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, 3, ...$$

λ is the rate per a specified, fixed interval

1.2 Investigating The Poisson Distribution

To get a feel for the Poisson distribution we can use our statistics calculator to plot a graph of the distribution for a simple example.

Example

The number of telephone calls received at a call centre during a busy weekday morning follows a Poisson distribution with a mean of 3 calls per five minute period.
Use your statistics calculator to plot a graph of the distribution.

For the Casio fx-991EX Classwiz this will require;

- MENU 7 Distribution
- ⇓ Scroll Down
- 2 Poisson PD
- 2 Variables

λ will always be 3 due to the "3 calls per five minute period"
Complete the following table for x = 0, 1, 2, 3, ...

With A Statistics Calculator

Check that for $x = 0$ you get a probability of 0.04978706837

Without A Statistics Calculator

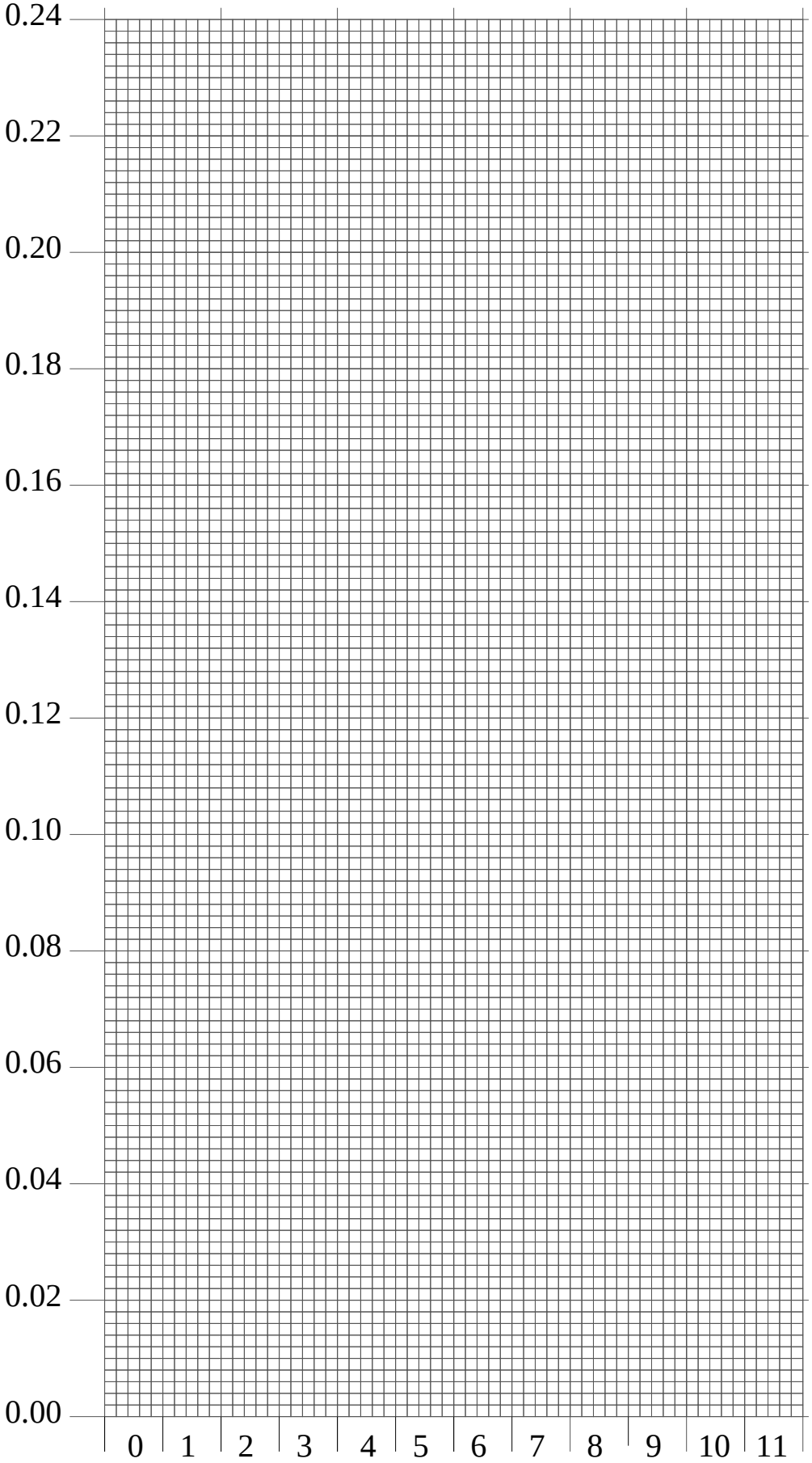
$$P(X = 0) = \frac{e^{-3} \times 3^0}{0!}$$

$$= 0.04978706837$$

Three Decimal Places

Either way, put the probability rounded to 3 decimal places into the table

$\lambda = 3$	x	$P(X = x)$
	$x = 0$	
	$x = 1$	
	$x = 2$	
	$x = 3$	
	$x = 4$	
	$x = 5$	
	$x = 6$	
	$x = 7$	
	$x = 8$	
	$x = 9$	


0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
1
2
3
4
5
6
7
8
9
10
11

1.3 Exercise

Question 1

$$X \sim Po\ (\ 4.5\ )$$

Determine $P\ (\ X = 7\ )$ by means of the following calculation;

$$P\ (\ X\ =\ 7\ )\ =\ \frac{e^{-4.5} \times 4.5^{7}}{7!}$$

Question 2

$$X \sim Po\ (\ 5.0\ )$$

Show the calculation needed to calculate $P\ (\ X = 4\ )$ directly from the definition of the Poisson distribution, which is;

If $X \sim Po\ (\ \lambda\ )$, then the Poisson distribution is given by

$$P\ (\ X\ =\ x\ )\ =\ \frac{e^{-\lambda}\ \lambda^{x}}{x!} \qquad x\ =\ 0,\ 1,\ 2,\ 3,\ ...$$

λ is the rate per a specified, fixed interval

Question 3

$$X \sim Po\ (\ 2.5\ )$$

Use the Poisson PD facility on your calculator to determine $P\ (\ X = 1\ )$

Question 4

$$X \sim Po\ (2.5)$$

Use the Poisson **C**D facility on your calculator to determine each of the following;

(i) $P(X \leqslant 5)$

(ii) $P(X > 2)$

(iii) $P(3 \leqslant X \leqslant 6)$

Question 5

$$X \sim Po\ (4.5)$$

Use the printed tables “Poisson Cumulative Distribution Function” to determine each of the following;

(i) $P(X = 2)$

(ii) $P(X \leqslant 1)$

(iii) $P(X \geqslant 4)$

(iv) $P(2 \leqslant X \leqslant 6)$

Question 6

$$X \sim Po\ (1.6)$$

Use either your statistics calculator or the printed tables "Poisson Cumulative Distribution Function" to determine each of the following;

(i) $P(X = 0)$

(ii) $P(X > 1)$

(iii) $P(X \leqslant 2)$

(iv) $P(1 < X \leqslant 3)$

Question 7

$$X \sim Po\ (8.0)$$

Use either your statistics calculator or the printed tables "Poisson Cumulative Distribution Function" to determine the values of a, b, c and d;

(i) $P(X \leqslant a) = 0.3134$

(ii) $P(X \leqslant b) = 0.7166$

(iii) $P(X < c) = 0.0996$

(iv) $P(X > d) = 0.8088$