Lesson 3

The Poisson Distribution
Further Statistics 1

3.1 Important Properties of the Poisson Distribution

A skill of a statistician is to know the properties of a variety of distributions, and so be able to deduce which is likely to model a given situation from the real world most effectively. The Poisson distribution has some very striking properties !

3.2 Mean and Variance

The Poisson distribution has the remarkable theoretical property that its mean is equal to its variance.

If

$$X \sim Po(\lambda)$$

Then

- $Mean\ of\ X = E(X) = \lambda$
- $Variance\ of\ X = Var(X) = \sigma^2 = \lambda$

The use of the letter *E* for the mean comes from “Expected Value”.
i.e. $E(X)$ is the value expected on average.

3.3 Example

Twenty sheets of titanium alloy were examined for surface flaws;

Nº of flaws x	Frequency f	xf	$x^2 f$
0	1		
1	6		
2	5		
3	3		
4	3		
5	1		
6	1		

(i) Calculate the mean and the variance of the data.

(ii) Suggest a suitable statistical distribution to model the number of flaws in a sheet of titanium alloy. Give a reason for your answer.

(iii) Use the statistical distribution you suggested in part (ii) to estimate the number of titanium alloy sheets that would contain 7 or more flaws in a batch of 1000 sheets.

3.4 Exercise

Question 1

The discrete random variable X has a Poisson distribution with mean 5.4
Find, giving your answer to 4 decimal places,

(i) $p\,(X = 2)$ **(ii)** $p\,(X \leqslant 3)$

(iii) $p\,(X > 4)$ **(iv)** $p\,(2 \leqslant X < 7)$

Question 2

In a factory it has been found, over a long period, that the number of accidents, X, reported per month has a Poisson distribution with parameter 1.5

(i) What is the mean number of accidents reported per year ?

(ii) What is the probability that in any given month, exactly two accidents are reported ? Give your answer to 4 decimal places.

(iii) What is the probability that, in a period of six months, fewer than ten accidents will be reported ? Give your answer to 4 decimal places.

Question 3

In Benson's roadside café, 40% of the customers buy a cup of tea.

(i) Benson is wondering, "What the probability of at least 4 of the next 10 customers will buy a cup of tea ?"
Use an appropriate distribution to answer Benson's question.

Benson has calculated that on a typical morning customer's arrive in the café at an average rate of 0.5 per minute

(ii) Find the probability that at least 10 customers arrive in the next 15 minutes.

(iii) Find the probability that exactly 10 customers arrive in the next 20 minutes.

(iv) Find the probability that in the next 20 minutes exactly 10 customers arrive and at least 4 of them buy a cup of tea.

Question 4

A book containing 278 pages has 182 misprints.
It is well known that misprints in books follow a Poisson distribution.
Find, correct to four decimal places, the probability that a particular page contains,

(i) no misprint

(ii) at least 4 misprints

(iii) not less than 2 misprints

Question 5

A supermarket is investigating the number of cherries in the fruit scones it sells. It uses a machine that can identify the cherries using x-rays, so the scones can still be sold. Here is the data;

Nº of cherries x	Frequency f	$x\,f$	x^2 f
0	82		
1	142		
2	44		
3	12		
4	5		
5	3		
6	2		

(i) Show the mean and the variance are approximately equal.

(ii) Tracy does not like cherries, and pulls seven out of a scone she has bought. What is the probability that a scone, chosen at random, contains seven cherries or more ? Give your answer to four decimal places.

Question 6

S2 Examination Question from June 2016, Q1

A student is investigating the number of cherries in a *Rays* fruit cake. A random sample of *Ray's* fruit cakes is taken and the results are shown in the table below,

N° of Cherries	0	1	2	3	4	5	⩾ 6
Frequency	24	37	21	12	4	2	0

(a) Calculate the mean and the variance of these data

[3 marks]

(b) Explain why the results in part (a) suggest that a Poisson distribution may be a suitable model for the number of cherries in a *Rays* fruit cake.

[1 mark]

The number of cherries in a *Rays* fruit cake follows a Poisson distribution with mean 1.5

A *Rays* fruit cake is to be selected at random

Find the probability that it contains

(c) **(i)** exactly 2 cherries

(ii) at least 1 cherry

[4 marks]

Rays fruit cakes are sold in packets of 5

(d) Show that the probability that there are more than 10 cherries, in total, in a randomly selected packet of Rays fruit cakes, is 0.1378 correct to four decimal places.

[3 marks]

Twelve packets of *Rays* fruit cakes are selected at random

(e) Find the probability that exactly 3 packets contain more than 10 cherries.

[3 marks]