

Lesson 4

The Poisson Distribution Further Statistics 1

4.1 Combining Poisson Distributions

A general result is going to be deduced by consider the following situation in two different ways;

The number of lorries passing an observation post north-bound along a single lane country road in a 15 minute interval is modelled by a Poisson distribution with mean 3. The number of other vehicles, mostly cars, passing the observation post in the same direction in a five minute interval is modelled by a Poisson distribution with mean 2. Find the probability that, exactly 3 vehicles, of any type, pass the observation point, north-bound in a 10 minute interval.

Method 1

Method 2

The fact that the same answer has been obtained via two different routes is suggestive of the fact that,

$$Po(2) + Po(4) = Po(2 + 4)$$

This is an illustration of a general result,

If two Poisson variables X and Y are independent, then the variable $Z = X + Y$ also has a Poisson distribution.

Furthermore, if

$$X \sim Po(\lambda)$$

$$\text{and } Y \sim Po(\mu)$$

then

$$X + Y \sim Po(\lambda + \mu)$$

4.2 Exercise

Question 1

The Monkey Finger YoYo Company accepts orders for their Prime41 professional YoYo by either telephone or through their website,

The mean number of sales per day are,

- Via Telephone : 23
- Via Website : 64

(i) What is the probability that the company receive over 100 orders in a day ?
Give you answer to four decimal places.

(ii) What are the two most likely number of orders to be received in a day ?

Question 2

Plans for a North-West Relief Road for Shrewsbury were announced in March 2019, along with £54 million of Government funding towards the £75 million project.

As part of the research into justifying the project the number of lorries passing along the town centre's Smithfield Road were counted.

The mean per hour was 36 travelling west and 27 travelling east.

Giving your answers to four decimal places, find the probability that in any given interval of 20 minutes,

(i) less than 8 lorries travelled east,

(ii) not more than 12 lorries travelled west,

(iii) a total of more than 30 lorries passed along the Smithfield Road,

Question 3

The number of emissions per minute from two different sources of radioactivity are modelled as independent Poisson random variables C and R .

First Source : 1 μg of Caesium, ^{137}Cs

The Caesium has a mean number of decays per second of 8.2×10^5

Second Source : 1 μg of Radium, ^{88}Ra

The Radium has a mean number of decays per second of 3.7×10^4

What is the probability of the total number of emissions from the two sources is no more than 860 decays per millisecond ?

Question 4

Tanya and Henry share a flat.

On average, on the six days each week when mail is delivered, Tanya receives a mean of 0.4 letters per day while Henry receives a mean of 0.3

- (i) Assuming their letters are independent, and that a Poisson distribution is appropriate, calculate the probability that on a particular day of the six each week when mail is delivered, exactly two letters arrive.
Give your answer accurate to four decimal places.
- (ii) Explain why it is not sensible to calculate probabilities for letters arriving per hour.

Question 5

Meteorites can be classified as being composed of either stone or iron.

It is estimated that on average, 665 stone meteorites, each with an individual mass that is greater than 10 grams. fall to Earth in any given week.

It is further estimated that on average 5460 iron meteorites, each with an individual mass greater than 10 grams fall to Earth a year.

Giving your answers to 4 decimal places, and assuming the arrivals may be modelled with a Poisson distribution, what is the probability in 24 hours that,

- (i) exactly 100 meteorites with a mass greater than 10 grams fall to Earth ?
- (ii) more than 90 meteorites with a mass greater than 10 grams fall to Earth ?

Question 6

Given that X and Y are independent random variables such that,

$$X \sim Po(2.5) \quad \text{and} \quad Y \sim Po(3.5)$$

find,

- (i) $p(X = 2 \text{ and } Y = 2)$ (ii) $p(X + Y = 5)$

Question 7

A secretary receives internal calls at a rate of 1 every 5 minutes and external calls at a rate of 2 every 5 minutes.

Calculate the probability that the total number of calls is,

- (i) 3 in a 4-minute period

- (ii) at least 2 in a 2-minute period

- (iii) no more than 5 in a 10 minute period

Question 8

During the working day the emails arriving to the account of a company director are classified into three types, personal, business and advertising.

Personal emails arrive at a mean rate of 1.8 per hour, business emails arrive at a mean rate of 3.7 per hour and advertising emails arrive at a mean rate of 1.5 per hour.

- (a) Find the probability that she receives,
 - (i) at least one of each type of email during a 30 minute period of the working day,

 - (ii) more than 50 emails in an 8-hour working day

- (b) Using a binomial model to find the probability that she receives more than 50 emails on exactly two days out of a 5-day working week.