Lesson 6

The Poisson Distribution
Further Statistics 1

6.1 Examination Questions

Question 1

One of the first applications of the Poisson distribution was in modelling the number of Prussian soldiers accidentally killed by horse-kick.
The data was gathered between 1875 and 1894 from ten Army Corps giving 200 pieces of data (20 years × Ten Corps).
During this time 122 soldiers were killed from a horse kick.

Nº of deaths x	Frequency f	xf	x^2 f
0	109		
1	65		
2	22		
3	3		
4	1		

(i) Show the mean and the variance are approximately equal.

(ii) Complete the following table to show how the actual frequency of deaths from horse-kick compares with that predicted by the Poisson distribution model with a suitable parameter derived from part (i).

N° of deaths	Actual Frequency	Poisson Predicted Frequency
0	109	
1	65	
2	22	
3	3	
4	1	

Question 2

Examination Question from 2017 'Sample Assessment Materials'

Two car hire companies hire cars independently of each other.

Car Hire A hires cars at a rate of 2.6 cars per hour.

Car hire B hires cars at a rate of 1.2 cars per hour.

(a) In a one-hour period, find the probability that each company hires exactly 2 cars.

[2 marks]

(b) In a one-hour period, find the probability that the total number of cars hired by the two companies is 3

[2 marks]

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9.

[2 marks]

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) **(i)** Find the mean of the number of cars with a defect

(ii) find the variance of the number of cars with a defect

[2 marks]

(e) **(i)** Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson distribution.

[2 marks]

Question 3

Examination Question from 2017 'Sample Assessment Materials'

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of a caller, chosen at random, being connected to the wrong agent is p.

The probability of at least 1 call in 5 consecutive calls being connected to the wrong agent is 0.049

The call centre receives 1000 calls each day.

(a) Find the mean and variance of the number of wrongly connected calls a day.

[7 marks]

(b) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent.

[2 marks]

(c) Explain why the approximation used in part (b) is valid.

[2 marks]

The probability that more than 6 calls each day are connected to the wrong agent using the binomial distribution is 0.8711 to 4 decimal places

(d) Comment on the accuracy of your answer in part (b).

[1 mark]

Question 4

Examination Question from May 2011 S2 Paper, Q2

A traffic officer monitors the rate at which vehicles pass a fixed point on a motorway. When the rate exceeds 36 vehicles per minute he must switch on some speed restrictions to improve traffic flow.

(a) Suggest a suitable model to describe the number of vehicles passing the fixed point in a 15 s interval.

[1 mark]

The traffic officer records 12 vehicles passing the fixed point in a 15 s interval.

(b) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not the traffic officer has sufficient evidence to switch on the speed restrictions.

[6 marks]

(c) Using a 5% level of significance, determine the smallest number of vehicles the traffic officer must observe in a 10 s interval in order to have sufficient evidence to switch on the speed restrictions.

[3 marks]

Question 5

Examination Question from 2018 'Sample Assessment Materials'

Tim and Sue are each typing a manuscript and they make errors at random.
On average, Tim makes 0.45 errors per page and Sue makes 0.2 errors per page.

A random sample of n pages of Tim's typing is taken.
The probability that these n pages contain no errors is less than 0.05

(a) Find the smallest possible value of n

[3 marks]

The random variable X represents the total number of errors in a random sample of 5 pages of Tim's typing and 5 pages of Sue's typing.

(b) Find $p(X = 2)$, stating a necessary assumption.

[3 marks]

Random samples, each consisting of 5 pages of Tim's typing and 5 pages of Sue's typing, are selected.

(c) Find the probability that in 10 of these samples there are at least 2 with no errors.

[4 marks]

Question 6

Examination Question from May 2013 S2 Paper, Q2

The number of defects per metre in a roll of cloth has a Poisson distribution with mean 0.25

Find the probability that,

(a) a randomly chosen metre of cloth has 1 defect

[2 marks]

(b) the total number of defects in a randomly chosen 6 metre length of cloth is more than 2

[3 marks]

A tailor buys 300 metres of cloth

(c) Using a suitable approximation find the probability that the tailor's cloth will contain less than 90 defects.

HINT : A suitable approximation is $N(\mu, \sigma^2)$ with $\mu = \sigma^2$
Don't forget the continuity correction !

[5 marks]

Question 7

Examination Question from May 2012 S2 Paper, Q4

The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2 per week.

(a) Find the probability that in the next four weeks the estate agent sells

(i) exactly 3 houses

(ii) more than 5 houses

[5 marks]

The estate agent monitors sales in periods of 4 weeks.

(b) Find the probability that in the next twelve of those 4 week periods there are exactly nine periods in which more than 5 houses are sold.

[3 marks]

The estate agent will receive a bonus if he sells more than 25 houses in the next 10 weeks.

(c) Use a suitable approximation to estimate the probability that the estate agent receives a bonus.

[6 marks]