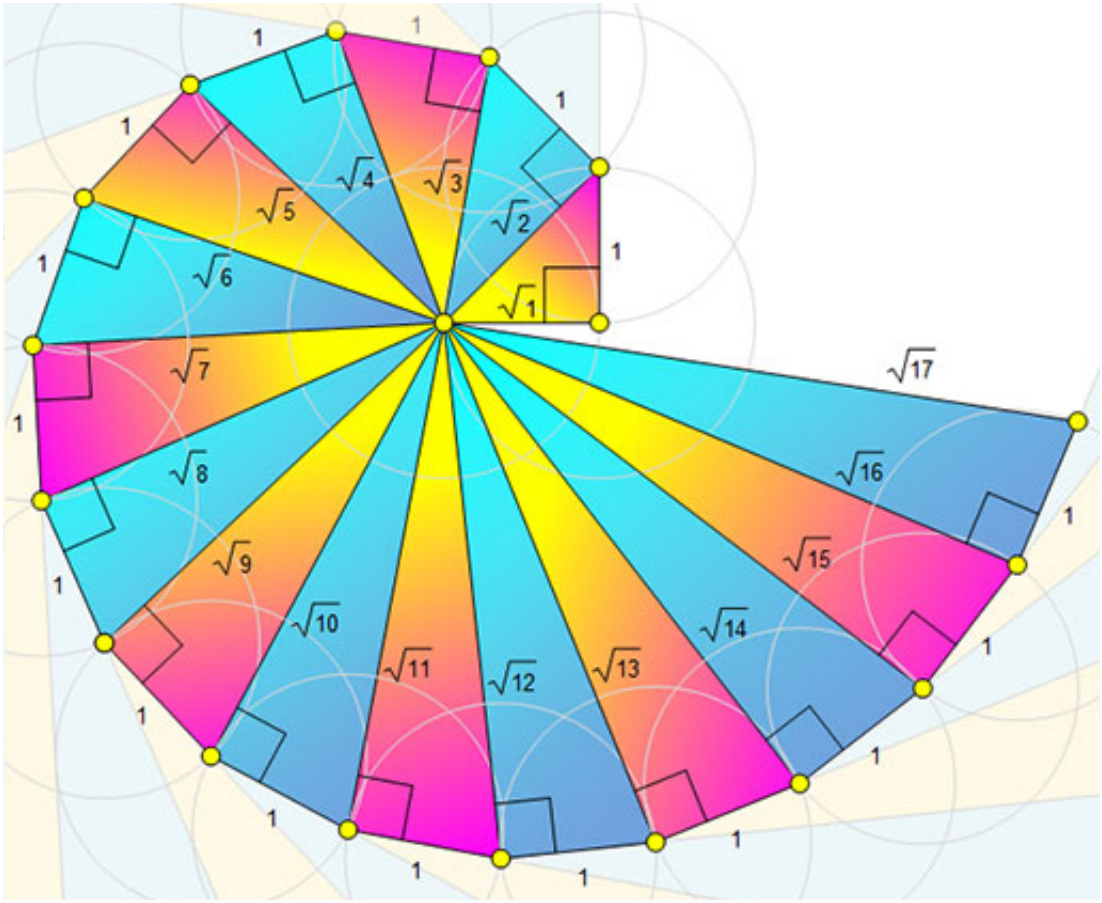
A-Level Mathematics
A-Level Further Mathematics
Pure Year 1

ALGEBRA

~ Surds and Indices II ~



Algebra

~ Surds and Indices ~

Lesson 1

A-Level Pure Mathematics : Year 1

Algebra of Surds and Indices II

1.1 Multiplication Involving Surds

Challenge #1

Find the exact value of,

$$\left(2 + 7\sqrt{2} - 5\sqrt{3}\right)\left(4 + 3\sqrt{2} + 2\sqrt{3}\right)$$

[6 marks]

Challenge #2

Find the exact value of,

$$\left(3\sqrt{2} + 4\sqrt{3} + 7\sqrt{6}\right)\left(5\sqrt{2} - 2\sqrt{3} + 3\sqrt{6}\right)$$

[6 marks]

1.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Expand and simplify, giving exact answers,

(i) $(5 + \sqrt{7})^2$

[2 marks]

(ii) $(2 + 5\sqrt{3})(4 - 2\sqrt{3})$

[3 marks]

(iii) $(3 + \sqrt{2})(3 - \sqrt{2})(2 + \sqrt{5})$

[4 marks]

(iv) $(6 + 2\sqrt{3} - \sqrt{5})(7 + \sqrt{3})$

[5 marks]

Question 2

A square-free number is an integer that is not divisible by any square number except 1.

(i) Which of the following are integers ?

$6 \quad \pi \quad \sqrt{8} \quad -4 \quad \sqrt{25} \quad 0.125 \quad \frac{4}{5} \quad \frac{81}{3}$

[1 mark]

(ii) Which of the following are square-free ?

7 121 50 12 6 99 65 147

[2 marks]

Question 3

Simplify each of the following, writing your answers in the form $a\sqrt{b}$ for integer values of a and b with b square free.

(i) $\sqrt{8}$

[1 mark]

(ii) $5\sqrt{20}$

[1 mark]

(iii) $\left(3\sqrt{2}\right)^3 - 4\sqrt{2}$

[2 marks]

(iv) $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

[2 marks]

Question 4

Show how to expand the brackets of

$$\left(1 + \sqrt{5}\right)\left(1 - \sqrt{5}\right)^3$$

to obtain an answer in the form $a + b\sqrt{5}$ where a and b are integers.

[3 marks]

Question 5

Find the exact value of,

$$\left(5 + 3\sqrt{2} + 6\sqrt{3}\right)\left(8 + 4\sqrt{2} - 2\sqrt{3}\right)$$

[6 marks]

Question 6

Which of the following numbers are square free ?

(i) $3 \times 5 \times 7$

(ii) $2 \times 3^2 \times 11$

(iii) 5×7^3

[2 marks]

Question 7

Explain why,

$$\sqrt{2 \times 3^3 \times 5^2} = 15\sqrt{6}$$

[2 marks]

Question 8

Simplify,

$$\sqrt{a^3 \times b^5}$$

where a and b are unknown integers.

Give your answer in the form $x\sqrt{y}$ where x and y are expressed in terms of a and b and y is square free.

[2 marks]

Question 9

The following number is too big for my calculator;

$$5^{52} \times 7 \times 13^{95}$$

Even so, square root this number, writing the answer in in the form $a\sqrt{p}$

where a & p are integers, that may be written in index form

and p is ☐ FREE.

[2 marks]

Question 10

Find the exact value of,

$$\left(7\sqrt{2} - 3\sqrt{5} + 4\sqrt{10}\right)\left(9\sqrt{2} + 2\sqrt{5} + 2\sqrt{10}\right)$$

[6 marks]

Question 11

Without using a calculator at any stage, devise a method that will determine the cube root of 1728

i.e. $\sqrt[3]{1728}$

[4 marks]