Lesson 2

A-Level Pure Mathematics : Year 1

Algebra of Surds and Indices II

2.1 Rationalising the Denominator

Mathematicians' dislike fractions which have any occurrence of a square root in the denominator. There are standard techniques for manipulating such fractions to remove all occurrences of an offending square root from the denominator.
This may well result in square roots in the numerator, but this is considered fine !

2.2 Example (GCSE Level 8, 9)

Consider the function,

$$f(a) = \frac{a}{a - \sqrt{a}} \qquad a \in \mathbb{Z},\ a \geqslant 2$$

(i) Carefully explain the restriction upon the domain of the function.

[3 marks]

(ii) Given, furthermore, that a is a square free number, rationalise the denominator of the function.
Simplify your answer.

[3 marks]

2.3 Example (A-Level)

(i) Give a definition of a rational number.

[2 marks]

(ii) Prove that, $\frac{\sqrt{5} - \sqrt{13}}{3\sqrt{5} + \sqrt{13}} = \frac{7}{8} - \frac{1}{8}\sqrt{65}$

[4 marks]

2.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Prove that, $\frac{3\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} = 13 - 2\sqrt{35}$

[4 marks]

Question 2

Prove that, $\frac{1}{(2 + \sqrt{3})^2} = 7 - 4\sqrt{3}$

[4 marks]

Question 3

Show that, $\dfrac{5}{\sqrt{75} - \sqrt{50}}$

can be written in the form $\sqrt{a} + \sqrt{b}$ where a and b are integers

[4 marks]

Question 4

A right angled triangle has an exact area of 9 cm^2

One side, not the hypotenuse, has an exact length of $(5 + \sqrt{7})$ cm

(i) Showing your method clearly, determine the exact length of the other side, not the hypotenuse, of the triangle.

[3 marks]

(ii) By using the Theorem of Pythagoras, determine the exact perimeter of the right angled triangle.

[3 marks]

Question 5

By first rationalising the denominators, then getting a common denominator, show that

$$\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{3}} = \frac{9\sqrt{2} + 10\sqrt{3}}{6}$$

[4 marks]

Question 6

Simplify

$$\frac{1}{\sqrt{a}} + \frac{4\sqrt{a}}{a}$$

where a is an integer.

Your final answer should have a denominator that is rational.

[5 marks]

Question 7

Solve the following equation, giving an exact answer,

$$8 - x\sqrt{12} = \frac{8x}{\sqrt{3}}$$

[5 marks]

Question 8

Given that $p = 3 - 2\sqrt{2}$ and $q = 2 - \sqrt{2}$ find the value of

$$\frac{p + q}{p - q}$$

Give your answer in the form $m + n\sqrt{2}$ where m and n are rational numbers that are to be found.

[5 marks]

Question 9

Showing clear working, evaluate the following expression giving an exact answer in as simple a form as possible,

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{2021}+\sqrt{2022}}$$

[7 marks]

Question 10

Write the following as a single fraction with a denominator that is rational,

$$\frac{7}{\sqrt{ab}} + \frac{b}{\sqrt{a}} + \frac{a}{\sqrt{b}}$$

where a and b are positive integers.

[6 marks]