Lesson 3

A-Level Pure Mathematics : Year 1
Algebra of Surds and Indices II

3.1 Equations Involving Surds & Indices

Example #1 : Index Matching
Given that

$$81\sqrt{3} = 3^a$$

find the value of a.

[2 marks]

Example #2 : Index Matching
Given that

$$\left(27x^{12}\right)^{\frac{5}{3}} = 3^a x^b$$

find the value of a and the value of b.

[2 marks]

Example #3 : Surd Coefficient Matching
Given that,

$$\left(3+\sqrt{c}\right)\left(2\sqrt{c}-3\right) = 1 + k\sqrt{c}$$

where c and k are prime numbers, find the value of c and the value of k

[3 marks]

Example #4 : Double Square Root Surds

Given that,

$$\sqrt{10 + 2\sqrt{21}} = \sqrt{a} + \sqrt{b} \qquad \text{with } a < b$$

find the value of a and the value of b, both of which are integers.

[3 marks]

3.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 55

Question 1

Without using a calculator, write down the value of each of the following.
Answers should be written as exact simplified rational numbers without indices.

(i) $25^{\frac{1}{2}}$ **(ii)** $27^{\frac{1}{3}}$ **(iii)** 3^{-2} **(iv)** $4^{\frac{3}{2}}$ **(v)** $9^{-\frac{1}{2}}$

(vi) $100^{-\frac{3}{2}}$ **(vii)** $81^{\frac{1}{4}}$ **(viii)** $81^{\frac{3}{4}}$ **(ix)** $81^{-\frac{1}{2}}$ **(x)** 81^{0}

[5 marks]

Question 2

Given that a, b and c are distinct prime numbers and that

$$y = 2^5 \times 3^4 \times 5^3$$

determine the value of $\sqrt{y}$

Write your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[2 marks]

Question 3

Without using a calculator, write down the value of each of the following. Answers should be written as simplified exact numbers without indices and with denominators that are rational.

(i) $\left(\frac{4}{7}\right)^{2}$ **(ii)** $\left(\frac{3}{2}\right)^{-2}$ **(iii)** $\left(\frac{22}{77}\right)^{0}$

(iv) $\left(\frac{3}{8}\right)^{-1}$ **(v)** $\left(\frac{50}{32}\right)^{\frac{1}{2}}$

[5 marks]

Question 4

Without using a calculator, write down the value of each of the following. Answers should be written as simplified exact numbers without indices and with denominators that are rational.

(i) $\left(-\frac{16}{54}\right)^{-\frac{1}{3}}$ **(ii)** $\left(\frac{9}{6\sqrt{2}}\right)^{3}$

(iii) $\left(\frac{25}{8}\right)^{\frac{1}{2}}$ **(iv)** $\left(\frac{7+\sqrt{5}}{8}\right)^{-1}$

[4 marks]

Question 5

Given that $8\sqrt{2} = 2^{a}$ find the value of a

[2 marks]

Question 6

Given that,

$$\sqrt{3 + 2\sqrt{2}} = \sqrt{a} + \sqrt{b} \qquad \text{with } a > b$$

find the value of a and the value of b, showing clear reasoning to justify your answer.

[3 marks]

Question 7

Given that,

$$\left(7 - \sqrt{c}\right)\left(4 + 2\sqrt{c}\right) = 6 + k\sqrt{c}$$

where c and k are integers and c is square free, find the value of c and the value of k

[3 marks]

Question 8

Showing all steps in your reasoning, work out the exact value of n, given that

$$\frac{1}{\sqrt[3]{9^4}} = 3^n$$

[2 marks]

Question 9

Given that

$$\left(a + \sqrt{5}\right)\left(3 + 2\sqrt{5}\right) = 31 + b\sqrt{5}$$

find the value of a and the value of b both of which are integers.

[3 marks]

Question 10

Given that,

$$x = \sqrt{6 + 2\sqrt{5}} - \sqrt{6 - 2\sqrt{5}}$$

prove that x has the value 2, exactly.

[4 marks]

Question 11

Given that for some prime, p,

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$

Find the value of m

[2 marks]

Question 12

Two composite numbers, g and h have prime number decompositions

$$g = a^3 \times b \times c^2 \qquad h = a \times b \times c^3$$

where a, b and c are distinct prime numbers.

(a) Express gh as a product of powers of its prime factors
Simplify your answer

[2 marks]

(b) Find the value of x, the value of y and the value of z given that,

$$\frac{g}{h} = a^x \times b^y \times c^z$$

[3 marks]

Question 13

Given that,

$$\sqrt{8 - 4\sqrt{3}} = \sqrt{a} - \sqrt{b} \qquad \text{with } a > b$$

find the value of a and the value of b, both of which are integers

[3 marks]

Question 14

GCSE Examination Question, January 2017, Paper 3H(R), Q18

Given that p is a prime number, rationalise the denominator of

$$\frac{7\sqrt{p} - p^2}{\sqrt{p^3}}$$

Simplify your answer

[3 marks]

Question 15

A-Level Examination Question, June 2019, Paper 2, Q1

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

[3 marks]

Question 16

Without using a calculator, and making your method clear, find the square root of

$$2^7 \times 3 \times 5^4$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[2 marks]

Question 17

Carefully showing your working, rationalise the denominator of,

$$\frac{1}{1 + \sqrt{2} + \sqrt{3}}$$

[4 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School
It may be freely duplicated and distributed, unaltered, for non-profit educational use
In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"
© 2025 Number Wonder
Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com