Lesson 11

A-Level Pure Mathematics : Year 2

Differentiation IV

11.1 Revision

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 50

Question 1

Differentiate each of the following, simplifying answers as appropriate;

(i) $y = \dfrac{2x^5}{15}$

[2 marks]

(ii) $y = ln\left(2x^3 + 7\right)$

[2 marks]

(iii) $y = \dfrac{5}{4x^2 - 3}$

[2 marks]

(iv) $y = e^{\sqrt{x}}$

[2 marks]

Question 2

The product rule states that $(uv)' = uv' + u'v$

Use the rule to differentiate $y = 7x^2 \cos x$

[3 marks]

Question 3

The quotient rule states that $\left(\frac{u}{v}\right)' = \frac{vu' - v'u}{v^2}$

Use the rule to differentiate the following, simplifying your answer;

$$y = \frac{ln(4x)}{x^2}$$

[3 marks]

Question 4

(i) Use derivatives of $sin\ x$ and $cos\ x$ to prove the derivative of $tan\ x$ is $sec^2\ x$

[4 marks]

(ii) Hence, or otherwise, use the chain rule to differentiate;

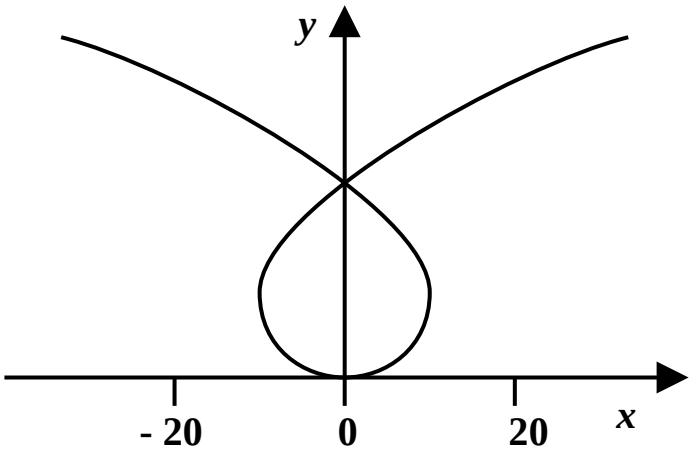
$$y = tan^2\ x$$

[2 marks]

Question 5

The graph is of the parametric equations;

$$x = 12t - t^3 \qquad \text{and} \qquad y = 3t^2$$



(i) Find, in terms of t,

(a) $\frac{dx}{dt}$ **(b)** $\frac{dy}{dt}$ **(c)** $\frac{dy}{dx}$

[2, 2, 2 marks]

(ii) Write down the coordinates of the point on the curve that corresponds to the parameter t having the value 1

[1 mark]

(iii) What is the gradient of the curve at your part (ii) point ?

[1 mark]

(iv) By making use of your part (ii) and (iii) answers, determine the equation of the tangent to the curve from the point at which $t = 1$

[2 marks]

Question 6

A curve has equation;

$$x^2 + 6xy - y^2 = 90$$

Find an expression for the gradient by means of implicit differentiation.

Write your answer in the form $\frac{dy}{dx} = f(x, y)$

[6 marks]

Question 7

The parametric equations of a curve are;

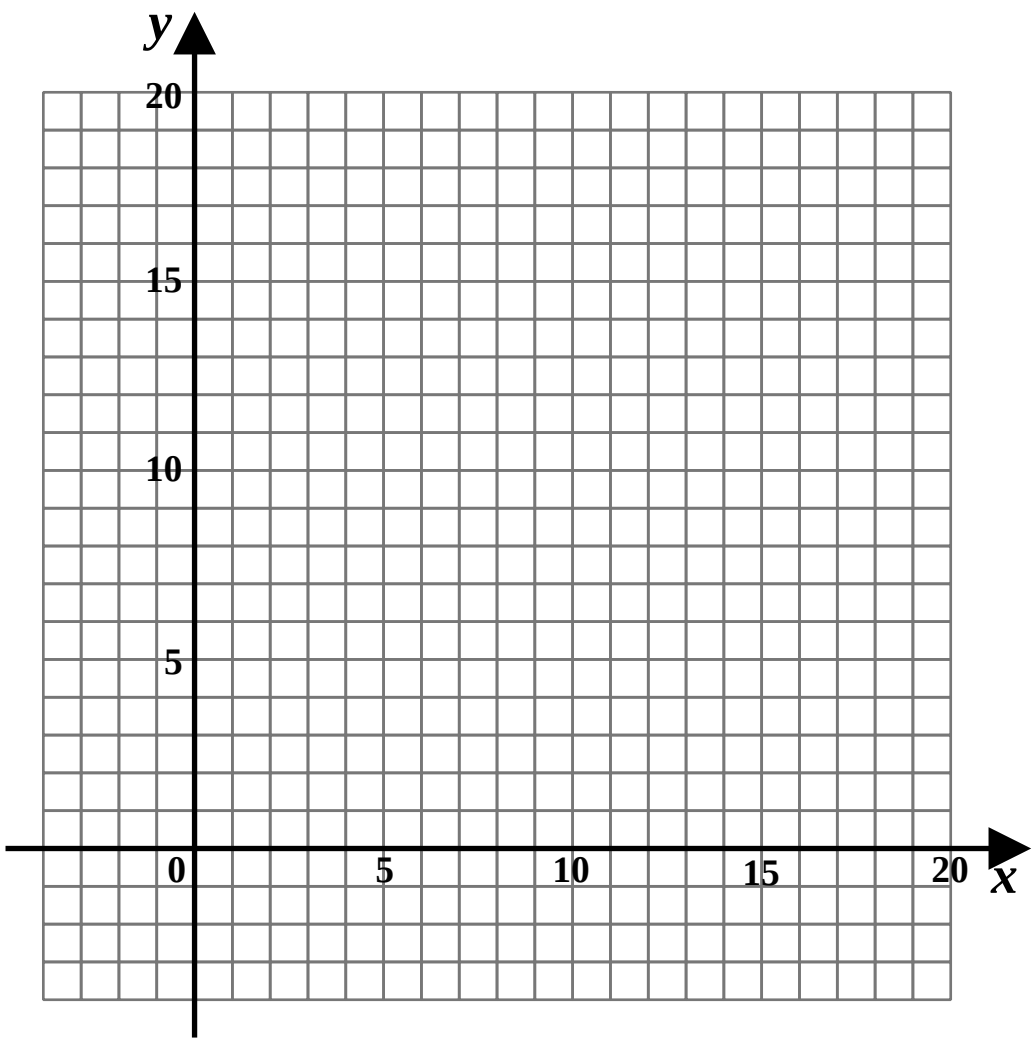
$$x = t^2 + t, \qquad y = t^2 - t$$

(i) Complete the following table by way of working out some points on the graph of this curve.

t	-4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4
x											
y											

[3 marks]

(ii) On the graph paper provided below plot the curve



[3 marks]

(iii) Find, in terms of t, an expression for the derivative of this curve.

[4 marks]

(iv) Find, in terms of x and y an expression for the derivative of this curve.

[4 marks]