A-Level Applied Mathematics
Year 2
Mechanics
Kinematics IV

V • A • R • I • A • B • L • E
A • C • C • E • L • E • A • T • I • O • N

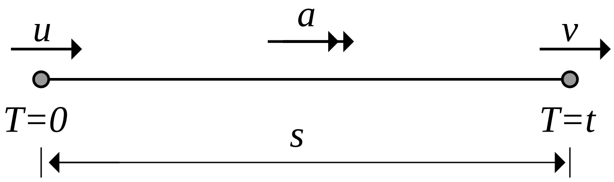
V A R I A B L E
A C C E L E R A T I O N
(Kinematics IV)

Lesson 1

A-Level Applied Mathematics : Mechanics : Year 2
Variable Acceleration (Kinematics IV)

1.1 Revisiting the five SUVAT equations

In Year 1 we considered a straight line interval over which a particle accelerated uniformly;



$s \ = \ \ displacement$

$u \ = \ initial \ velocity$

$v \ = \ final \ velocity$

$a \ = \ \ acceleration \ (constant)$

$t \ = \ time$

$$v \ = \ u \ + \ at \qquad\qquad s$$

$$s \ = \ vt \ - \ \frac{1}{2}\, a\, t^2 \qquad\qquad u$$

$$s \ = \ ut \ + \ \frac{1}{2}\, a\, t^2 \qquad\qquad v$$

$$s \ = \ \left(\frac{v \ + \ u}{2} \right) t \qquad\qquad a$$

$$v^2 \ = \ u^2 \ + \ 2as \qquad\qquad t$$

In Year 1, in Kinematics II, these *suvat* equations were applied to projectiles launched vertically upwards. In Year 2, in Kinematics III, this topic was extended to tackle first a projectile launched horizontally, then at any angle. A key idea was that what happens in the *x* direction can be analysed separately from what happens in the *y* direction.

Furthermore, the parabolic flight path of a projectile was determined to be,

$$y \ = \ x \tan\theta \ - \ \frac{g\, x^2}{2\, u^2} \left(1 \ + \ \tan^2\theta \right)$$

where *g* is the magnitude of the acceleration due gravity, 9.8 m s-and (x, y) is a coordinate on the flight path. This equation is worth memorising, although its derivation from the *suvat* equations may be asked for in an examination (See Lesson 4 of Kinematics III for the proof).

1.2 Constant Acceleration Vector Questions

Consider an ice-skater moving about a flat ice rink. East can be considered to be the *x*-axis direction and North the *y*-axis direction. For the sake of compactness, the vector **i** can represent the words "*x*-axis direction" and the vector **j** the words "*y*-axis direction." As **i** and **j** are assigned a length of 1 unit, they are known as **unit vectors** in the *x* and *y* directions respectively.

With a projectile, the horizontal *x*-axis had zero acceleration and the vertical *y*-axis had non-zero constant acceleration. However, with our ice-skater, and the altered understanding of what the *x*-axis and the *y*-axis represent, there could be constant non-zero acceleration in both directions. As before, each direction can be analysed independently of the other, but now the full *suvat* equations may need applying in both directions. **Vector techniques** are used to make this simple to implement in practice as illustrated by the following example.

1.3 Example

An ice-skater accelerates uniformly at $(0.5\mathbf{i} + \mathbf{j})$ m s^{-2} for 4 seconds.

She is initially at $(3\mathbf{i} - 7\mathbf{j})$ metres from a fixed origin O and moving with velocity $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1}

Where is she after the four seconds have elapsed ?

1.4 Exercise

Take **i** and **j** to be unit vectors due east and north respectively.

Question 1

A sand yacht is initially $(13\mathbf{i} - 27\mathbf{j})$ metres from a fixed origin O, moving with velocity $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1} and accelerating uniformly.

Six seconds later it is $(-11\mathbf{i} + 9\mathbf{j})$ metres from O.

(i) What is the acceleration of the sand yacht ?

Give your answer in the form $(a_x\mathbf{i} + a_y\mathbf{j})$ m s^{-2} where a_x and a_y are constants whose values you must state.

(ii) What is the velocity of the sand yacht at the end of the six seconds ?

(iii) Give your part (ii) answer as a speed in m s^{-1} and a bearing.

Question 2

A motor boat passes a buoy located 120 metres East and 70 metres North of a harbour entrance. It's velocity as it passes the buoy is $(4\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

It accelerates uniformly at $(-0.3\mathbf{i} + 0.2\mathbf{j})$ m s^{-2}

Twenty seconds later,

(i) how far is the motor boat from the harbour entrance ?

(ii) what is the motor boat's speed ?

(iii) on what bearing is the motor boat heading ?

Question 3

Particle P has an initial velocity of $(3\mathbf{i} - 7\mathbf{j})$ m s^{-1} and a constant acceleration of $(-\mathbf{i} + \mathbf{j})$ m s^{-2}

(i) Find an expression for the velocity at time t.

[2 marks]

(ii) What is the speed of the particle when $t = 4$ seconds ?

[2 marks]

(iii) At what time is the particle travelling south-west ?

[3 marks]

(iv) At what time is the speed of the particle equal to $2\sqrt{2}$ m s^{-1} ?

[5 marks]

Question 4

A particle A starts at the point with position vector $(12\text{i} + 12\mathbf{j})$ metres.
The initial velocity of A is $(-\mathbf{i} + \mathbf{j})$ m s^{-1}
It has constant acceleration $(2\mathbf{i} - 4\mathbf{j})$ m s^{-2}
Particle, B, has initial velocity $\mathbf{i}$ m s^{-1} and constant acceleration $2\mathbf{j}$ m s^{-2}
After 3 seconds the two particles collide.
Find,

(i) the speeds of the two particles when they collide

(ii) the position vector of the point where the two particles collide

(iii) the position vector of B's starting point

Question 5

A boat moves, with constant acceleration from the origin, so that at time t its velocity is given by the expression,

$$\boldsymbol{v} = 2(7 - t)\mathbf{i} + 4(3 - t)\mathbf{j}$$

(i) Find the time when the boat is heading due South.

(ii) Find, by writing the velocity in the form,

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

(a) the initial velocity of the boat

(b) the acceleration of the boat

(iii) Find the distance between the origin and the boat when t = 10 seconds