Lesson 2

A-Level Applied Mathematics : Mechanics : Year 2

Variable Acceleration (Kinematics IV)

2.1 Constant Acceleration Vector Examination Questions

These questions are rather good at testing your understanding of a mechanical system and your ability to then "bend" the mathematics to model the situation. This is very different to pure mathematics, where you place far more trust in the mathematics to guide you to through a problem to a logical conclusion.

In our introductory example, you are going to be told that there is a time at which a racing yacht is moving in a direction $2\mathbf{i} - 5\mathbf{j}$. This is telling you the direction of the velocity vector but not its magnitude. To handle this algebraically, write the velocity as $\lambda(2\mathbf{i} - 5\mathbf{j})$ where λ is an unknown constant.

2.2 Example

A racing yacht moves with constant acceleration $(\mathbf{i} - 2\mathbf{j})$ m s^{-2}
At time $t = 0$, the yacht is at the point A with velocity $(-3\mathbf{i} + \mathbf{j})$ m s^{-1}
At time $t = T$ seconds, it is moving in the direction of vector $(2\mathbf{i} - 5\mathbf{j})$

(i) Find the value of T

[3 marks]

(ii) Find the velocity of the yacht, in km/h at time T

[3 marks]

2.3 Exercise

Assume **i** and **j** are horizontal unit vectors due east and due north respectively.

Question 1

Photograph by Darren Heath, 2017

A experimental racing yacht moves with constant acceleration $(2\mathbf{i} - 7\mathbf{j})$ m s^{-2}
At time $t = 0$, the yacht is at the point A, moving with velocity $(-3\mathbf{i} + 5\mathbf{j})$ m s^{-1}
At time $t = T$ seconds, Y is moving in the direction of vector $(\mathbf{i} - 4\mathbf{j})$

(i) Find the value of T

[3 marks]

(ii) Find the velocity of the yacht, in km/h, at time T

[3 marks]

Question 2

A-Level Examination Question from June 2018, Q8

A particle P moves with constant acceleration. At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})$ metres.

(a) Show that the magnitude of the acceleration of P is 2.5 m s^{-2}

[4 marks]

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})$ m s^{-2}

At the instant when P reaches the point B, the direction of P is north east.

(b) Find the time it takes for P to travel from A to B.

[4 marks]

Question 3

A-Level Examination Question from June 2019, Q2

A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})$ m s^{-2}

At time $t = 0$, the particle is at the point A, moving with velocity $(-\mathbf{i} + 4\mathbf{j})$ m s^{-1}

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T

[4 marks]

At time $t = 4$ seconds, P is at the point B

(b) Find the distance AB

[4 marks]

Question 4

Old Examination Question

A ship is moving such that at time 12:00 its position is O and its velocity is $(-4\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

At 14:00, the ship is travelling with velocity $(-2\mathbf{i} - 6\mathbf{j})$ km h^{-1}. Relative to O, the ship has displacement $\boldsymbol{s}$ at time t hours after 12:00, where $t \geqslant 0$.

Modelling the ship as a particle with constant acceleration, find:

(a) the acceleration of the ship

[2 marks]

(b) an expression for $\boldsymbol{r}$ in terms of t

[2 marks]

(c) the time at which the ship is directly south-west of O

[3 marks]

At time t hours after 12:00, another ship has displacement $\boldsymbol{r} = (40 - 25t)\mathbf{j}$ relative to O.

(d) Find the position vector of the point where the two ships meet.

[4 marks]