Lesson 4

A-Level Applied Mathematics : Mechanics : Year 2

Variable Acceleration (Kinematics IV)

4.1 Differentiation and Variable Acceleration

The A-Level examination may includes a straight forward variable acceleration question involving only differentiation. As an example here is such a question from the June 2023 Mechanics paper.

4.2 Example

A-Level Examination Question from 20th June 2023, Paper 32, Q3 (Edexcel)

At time t seconds, where $t \geqslant 0$, a particle P has velocity $\boldsymbol{v}$ m s^{-1} where

$$\boldsymbol{v} = \left(t^2 - 3t + 7\right)\mathbf{i} + \left(2t^2 - 3\right)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$

[3 marks]

(b) the value of t when P is moving parallel to ($\mathbf{i} + \mathbf{j}$)

[2 marks]

(c) the acceleration of P at time t seconds

[2 marks]

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$

[2 marks]

4.3 Exercise

Question 1

A particle, initially at the origin, has position vector $\boldsymbol{r}$ where;

$$\boldsymbol{r} = t^3\,\mathbf{i} + 4t^2\,\mathbf{j}$$

(i) What is the particle's position when t = 3 seconds ?
Give your answer in the form $p\,\mathbf{i} + q\,\mathbf{j}$

(ii) How far is the particle from the origin when t = 3 seconds ?

(iii) Find an expression for the particle's velocity, $\boldsymbol{v}$, in terms of t.
Give your answer in the form $\boldsymbol{v} = p(t)\,\mathbf{i} + q(t)\,\mathbf{j}$

(iv) What is the particle's velocity when t = 3 seconds ?
Give your answer in the form $p\,\mathbf{i} + q\,\mathbf{j}$

(v) Give the particle's speed when t = 3 seconds.

(vi) Find an expression for the particle's acceleration, $\boldsymbol{a}$, in terms of t.
Give your answer in the form $\boldsymbol{a} = p(t)\,\mathbf{i} + q(t)\,\mathbf{j}$

(vii) What is the particle's acceleration when t = 3 seconds ?
Give your answer in the form $p\,\mathbf{i} + q\,\mathbf{j}$

(viii) Give the magnitude of the particle's acceleration when t = 3 seconds.

(ix) Why could the *suvat* equations not be used in answering the above ?

Question 2

A particle's position is given by;

$$\boldsymbol{r} = \left(3t^2 - 9\right)\mathbf{i} + \left(5t^2 + 3t\right)\mathbf{j}$$

(i) Find an expression for the particle's velocity, $\boldsymbol{v}$, in terms of t.

(ii) Find an expression for the particle's acceleration, $\boldsymbol{a}$, in terms of t.

(iii) Could you use the *suvat* equations to investigate this particle's motion ? Explain your answer.

Question 3

M2 Examination question from January 2006, part of Q2

A particle P is moving so that its position vector $\boldsymbol{r}$ metres at time t seconds is given by

$$\boldsymbol{r} = \left(t^2 + 4t\right)\mathbf{i} + \left(3t - t^3\right)\mathbf{j}$$

Calculate the speed of P when $t = 3$

[5 marks]

Question 4

A particle moves so that at time t seconds its position vector, $\boldsymbol{r}$ metres, relative to a fixed origin O is given by;

$$\boldsymbol{r} = \left(2t^2 - pt\right)\mathbf{i} + \left(t^3 - 3t\right)\mathbf{j}$$

where p is a constant.

(i) Find an expression for the velocity, $\boldsymbol{v}$, in terms of t.

(ii) Given that the particle comes to instantaneous rest, find the time at which it is at rest and hence the value of p.

Question 5

Relative to a fixed origin O, the position vector of a particle P at time t seconds is $\boldsymbol{r}$ metres, where,

$$\boldsymbol{r} = 16\sqrt{t}\,\mathbf{i} - \frac{1}{t}\,\mathbf{j}$$

Where is the particle when it is heading north-east ?

Question 6

A particle moves in a horizontal plane. At time t seconds, the position vector of P is $\boldsymbol{r}$ metres relative to a fixed origin O where,

$$\boldsymbol{r} = \left(18t - 4t^3\right)\mathbf{i} + ct^2\mathbf{j}, \quad t \geqslant 0$$

and where c is a positive constant.

When $t = 1.5$, the speed of P is 15 m s^{-1}

(i) Determine the value of c

[6 marks]

(ii) Find the acceleration of P when $t = 1.5$

[3 marks]

Question 7

A particle moves so that its position vector, in metres, relative to a fixed origin O at time t seconds is

$$\boldsymbol{r} = \left(2t^2 - 3\right)\mathbf{i} + \left(7 - 4t\right)\mathbf{j} \quad \text{where } t \geqslant 0$$

(a) Show that the particle is north-east of O when $t^2 + 2t - 5 = 0$

[2 marks]

(b) Hence determine the distance of the particle from O when it is north-east of O, giving your answer correct to 3 significant figures.

[3 marks]

A second particle moves with constant acceleration $\left(3a\,\mathbf{i} - 2a\,\mathbf{j} \right)$ m s^{-1}.

When $t = 0$ the velocity of the particle is $(5\mathbf{i} + 6\mathbf{j})$ m s^{-1} and its position vector relative to O is $5\mathbf{j}$ m.

When $t = 2$ seconds the particle is travelling with velocity $(b\,\mathbf{i} + 2b\,\mathbf{j})$ m s^{-1}.

(c) Find the speed and direction of the particle when $t = 2$

[6 marks]

(d) Find the distance between the two particles at this time.

[4 marks]